

Econ 211

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Motivation

- ▶ Last lecture, saw that
 - ▶ people appear relatively patient when all outcomes are in the future
 - ▶ people appear relatively impatient when trading off between right now and the future
- ▶ Standard exponential discounting model does not allow for this
 - ▶ When trading off consumption between consecutive periods, discount factor is always δ
- ▶ We need a behavioral model which gives treats the present differently than all future periods

The $\beta - \delta$ Discounting Model

- ▶ First proposed by Strotz (1955) and popularized by Laibson (1997)
- ▶ Recall consumption stream $c = (c_1, c_2, \dots, c_T)$
- ▶ Utility function:

$$\begin{aligned}U(c) &= u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3) + \dots + \beta\delta^{T-1} u(c_T) \\&= u(c_1) + \beta \left[\delta u(c_2) + \delta^2 u(c_3) + \dots + \delta^{T-1} u(c_T) \right]\end{aligned}$$

where

- ▶ $0 \leq \beta \leq 1$
- ▶ $0 \leq \delta \leq 1$
- ▶ Model known as β - δ (*beta-delta*) *discounting*
- ▶ Embeds the standard exponential discounting model if $\beta = 1$
- ▶ All periods that are not the present are additionally discounted by β , so present period gets a relative boost
 - ▶ We say that this kind of preference has *present-bias*

Example: Present-Bias Leads to Time-Inconsistency

- ▶ Three periods: $t = 1, 2, 3$
- ▶ In period 2, choose what to eat
- ▶ Two options:
 1. Eat well: $u_2 = 5, u_3 = 10$
 2. Eat poorly: $u_2 = 8, u_3 = 6$
- ▶ Assume that DM has beta-delta preferences with $\beta = \frac{1}{2}, \delta = 1$
- ▶ Decision in period 1?
 - ▶ Eat well: $U = \frac{1}{2}5 + \frac{1}{2}10 = 7.5$
 - ▶ Eat poorly: $U = \frac{1}{2}8 + \frac{1}{2}6 = 7$
 - ▶ Decision: eat well
- ▶ Decision in period 2:
 - ▶ Eat well: $U = 5 + \frac{1}{2}10 = 10$
 - ▶ Eat poorly: $U = 8 + \frac{1}{1}6 = 11$
 - ▶ Decision: eat poorly

Procrastination

- ▶ In many economic problems, agent must do a task
 - ▶ Task needs to be done exactly once
 - ▶ Agent Has several time periods to do task
- ▶ To analyze these types of decisions, use *backwards induction*: start analysis at the end of the process and work back to the first period
 - ▶ *Naive* agent is time inconsistent, but assumes self will be time-consistent in future
 - ▶ *Sophisticated* agent is time inconsistent, and knows self will be time-inconsistent in future

Example: Paper Writing

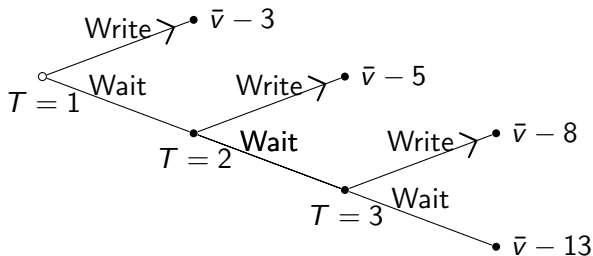
- ▶ Suppose student has a paper due in 4 weeks
- ▶ Can write the paper on weekend 1, 2, 3, or 4
- ▶ Cost of writing paper is missing going to movies with friends:
 - ▶ Weekend 1: bad movie, cost = 3
 - ▶ Weekend 2: OK movie, cost = 5
 - ▶ Weekend 3: good movie, cost = 8
 - ▶ Weekend 4: great movie, cost = 13
- ▶ Benefit of writing the paper is $\bar{v} > 0$, received in week 5 when grades are given
- ▶ For all types of agents, assume $\delta = 1$ in what follows
- ▶ For time-inconsistent types, assume $\beta = \frac{1}{2}$

When Does Time-Consistent Agent Write Paper?

- ▶ Decision in week 4:
 - ▶ Have to do the paper at this point, so no real choice to be made
- ▶ Decision in week 3:
 - ▶ If do paper, utility is $\bar{v} - 8$
 - ▶ If wait till next week, utility is $\bar{v} - 13$
 - ▶ So, will do paper in week 3 (if not done already)
- ▶ Decision in week 2:
 - ▶ If do paper, utility is $\bar{v} - 5$
 - ▶ If don't write paper, know will write next week for utility $\bar{v} - 8$
 - ▶ So, will do paper in week 2 (if not done already)
- ▶ Decision in week 1:
 - ▶ If do paper, utility is $\bar{v} - 3$
 - ▶ If don't write paper, know will write next week for utility $\bar{v} - 5$
 - ▶ So, will do paper in week 1

Decision Tree

- ▶ Helpful to keep track of decisions of agent with a decision tree
- ▶ Note decision each period is *whether* to write or wait, not *when* to write



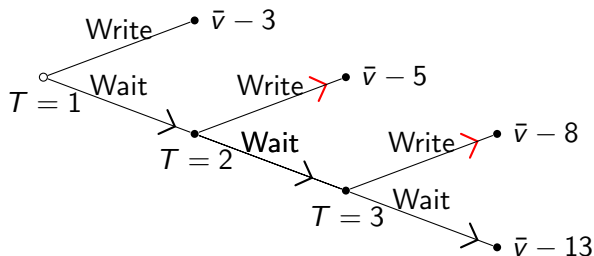
When Does Naive Time-Inconsistent Agent Write Paper?

- ▶ Decision in week 4:
 - ▶ Have to do the paper at this point, so no real choice to be made
- ▶ Decision in week 3:
 - ▶ If do paper, utility is $\frac{1}{2}\bar{v} - 8$
 - ▶ If wait till next week, utility is $\frac{1}{2}\bar{v} - \frac{1}{2}13 = \frac{1}{2}\bar{v} - 6.5$
 - ▶ So, will choose NOT to do paper in week 3
- ▶ Decision in week 2:
 - ▶ If do paper, utility is $\frac{1}{2}\bar{v} - 5$
 - ▶ If don't write paper:
 - ▶ Remember, thinks future self is time-consistent
 - ▶ So, thinks (incorrectly!) that will do paper in week 3
 - ▶ From perspective of week 2, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}8 = \frac{1}{2}\bar{v} - 4$
 - ▶ So, will choose NOT to do paper in week 2

Naive Time-Inconsistent Agent, con't

- ▶ Decision in week 1:
 - ▶ If do paper, utility is $\frac{1}{2}\bar{v} - 3$
 - ▶ If don't write paper:
 - ▶ Remember, thinks future self is time-consistent
 - ▶ So, thinks (incorrectly!) that will do paper in week 2
 - ▶ From perspective of week 1, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}5 = \frac{1}{2}\bar{v} - 2.5$
 - ▶ So, will choose NOT to do paper in week 1
- ▶ Overall result:
 - ▶ Agent waits until week 4 to do paper, misses best movie

Decision Tree for Naive Agent



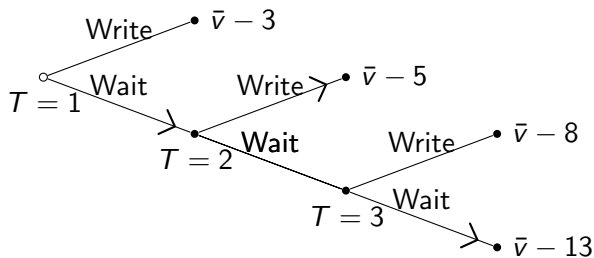
When Does Sophisticated Time-Inconsistent Agent Write Paper?

- ▶ Decision in week 4:
 - ▶ Have to do the paper at this point, so no real choice to be made
- ▶ Decision in week 3:
 - ▶ If do paper, utility is $\frac{1}{2}\bar{v} - 8$
 - ▶ If wait till next week, utility is $\frac{1}{2}\bar{v} - \frac{1}{2}13 = \frac{1}{2}\bar{v} - 6.5$
 - ▶ So, will choose NOT to do paper in week 3
- ▶ Decision in week 2:
 - ▶ If do paper, utility is $\frac{1}{2}\bar{v} - 5$
 - ▶ If don't write paper:
 - ▶ Remember, knows future self is time-inconsistent
 - ▶ So, thinks (correctly) that will NOT do paper in week 3
 - ▶ From perspective of week 2, utility of waiting is $\frac{1}{2}\bar{v} - 6.5$
 - ▶ So, will choose to DO paper in week 2

Sophisticated Time-Inconsistent Agent, con't

- ▶ Decision in week 1:
 - ▶ If do paper, utility is $\frac{1}{2}\bar{v} - 3$
 - ▶ If don't write paper:
 - ▶ Remember, knows future self is time-inconsistent
 - ▶ So, thinks (correct) that will do paper in week 2
 - ▶ From perspective of week 1, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}5 = \frac{1}{2}\bar{v} - 2.5$
 - ▶ So, will choose NOT to do paper in week 1
- ▶ Overall result:
 - ▶ Agent waits until week 2 to do paper, meaning agent misses OK movie

Decision Tree for Sophisticated Agent



Measuring $\beta - \delta$ Preferences

- ▶ Again, assume for simplicity that $u(x) = x$
- ▶ Suppose you say that you are indifferent between \$100 in one month and \$Y in two months
- ▶ Then we must have $\beta\delta 100 = \beta\delta^2 Y$, which implies

$$\delta = \frac{100}{Y}$$

- ▶ Suppose additionally you are indifferent between \$100 today and \$X in one month
- ▶ Then we must have $100 = \beta\delta X$
- ▶ Together with the equation for δ above, this implies

$$\beta = \frac{Y}{X}$$

More Details on Multiple Price List Methodology

- ▶ Most commonly used experimental method to estimate individual time preferences
- ▶ Choices between a smaller, sooner reward and a later, larger reward
- ▶ Typically one option stays fixed while the other varies
- ▶ Point at which subject switches from smaller/sooner reward to larger/later reward helps estimate their time preference parameters
- ▶ Essential to have both delay = 0 and delay > 0 lists to separately identify β and δ

Wrap-up

- ▶ No required reading for this lecture
- ▶ However, post in discussion forum your idea for an original research study
 - ▶ Can be experiment (lab or field) or observational study
 - ▶ Must be more than straight replication of existing study we've seen so far in class
 - ▶ Describe design, subject population, and expected results
 - ▶ Please also give feedback on at least one classmate's idea