Econ 211

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The Standard Model: Exponential Discounting

Motivation

- Which would you rather have?
  - \$100 today OR \$95 one month

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- If you value money today more than the same amount of money in the future, then we say you are *impatient*

Consumption Over Time

- Stream of consumption (or wealth or income) over T time periods, starting with period 1:

$$c = (c_1, c_2, c_3, \ldots, c_T)$$

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- If impatient, then would prefer to have an extra dollar today rather than tomorrow, implying

$$\frac{\partial U}{\partial c_t} > \frac{\partial U}{\partial c_{t+1}}$$

  or equivalently:

$$\frac{\frac{\partial U}{\partial c_{t+1}}}{\frac{\partial U}{\partial c_t}} < 1$$

Exponential Discounting Model

- A popular model used in many fields of economics to represent impatient preferences
- Utility of consumption stream c:

$$\begin{aligned} U(c) &= \sum_{t=1}^{T} \delta^{t-1} u(c_t) \\ &= u(c_1) + \delta u(c_2) + \delta^2 u(c_3) + \ldots + \delta^{T-1} u(c_T) \end{aligned}$$

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- δ is called *discount factor*, where $0 < \delta < 1$
- Consumer will choose consumption stream that maximizes $U(c)$
- Is this utility function is impatient? Yes, since

$$\frac{\frac{\partial U}{\partial c_{t+1}}}{\frac{\partial U}{\partial c_t}} = \frac{\delta \frac{\partial u}{\partial c_{t+1}}}{\frac{\partial u}{\partial c_t}} = \delta < 1$$

- For δ closer to 0, agent becomes *more impatient*

Example: Doing Your Laundry

- Suppose your utility each day is proportional to how many clean outfits you have to wear
- On Friday that you have just 2 clean outfits left
- You can do laundry on Friday, Saturday, or Sunday, or Monday
- Doing laundry is annoying: -5 utils the day you choose to do it
- Doing laundry gets you 5 clean outfits, but you use one each day
- In summary:

	Utility on day			
	F	Sa	Su	M
Do laundry Fri	-3	5	4	3
Do laundry Sat	2	-4	5	4
Do laundry Sun	2	1	-5	5
Do laundry Mon	2	1	0	-5

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Saturday?	2	$+\delta(-4)$	$+\delta^2(5)$	$+\delta^3(4)$
Sunday?	2	$+\delta(1)$	$+\delta^2(-5)$	$+\delta^3(5)$
Monday?	2	$+\delta(1)$	$+\delta^2(0)$	$+\delta^3(-5)$

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- Utilities under various values of δ:

	Total utility if $\delta =$			
	1	0.6	0.52	0.25
Do laundry Fri	9*	2.09	1.10	-1.45
Do laundry Sat	7	2.27*	1.83	1.38
Do laundry Sun	3	1.88	1.87*	2.02
Do laundry Mon	-2	1.52	1.82	2.17*

Checking Follow Through

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- Will you follow through with plan? Yes, since utility of doing on Saturday is still highest

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- Any exponential-discounting decision maker will be time-consistent

How Do We Measure Time Preferences?

- Suppose you are indifferent between \$100 today and \$X in one month
- Utility of \$100 today: $u(\$100)$
- Utility of \$X next month: $\delta u(\$X)$ (assuming monthly discount factor)
- Thus we must have $u(\$100) = \delta u(\$X)$, which implies

$$\delta = \frac{u(\$100)}{u(\$X)}$$

- If we make the assumption that $u(x) = x$, then

$$\delta = \frac{100}{X}$$

- Thus we can estimate time preferences by looking at switch point on price list

Evidence of Time Inconsistency

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  - That is, consumers make a different choice for tomorrow's consumption when asked today vs when asked tomorrow
  - Such consumers will have a *self-control problem*
- Also, we see that some people are aware of their time inconsistency
  - A *naive* agent believes (incorrectly) that he will follow through on his plans
  - A *sophisticated* agent knows that she may not follow through, so she may look for ways to *commit* herself to the plan

Lab Evidence: McClure et at (2007)

- Subjects told to come into the lab thirsty
- Experiment lasts at least 30 minutes
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- Treatment 1 (immediate): choose either
  - 1 juice now (early) OR
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- Treatment 2 (delay): choose either
  - 1 juice in 20 minutes (early) OR
  - 2 juices in 25 minutes (later)
- Subjects know this is their only chance to get a drink during the experiment

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  - Treatment 1 (immediate): 60% choose early option
  - Treatment 2 (delay): 30% choose early option

Field Evidence: Read, Loewenstein, and Kalyanaraman (1999)

- Subjects get vouchers from certain movies off of a list
- List includes “high brow” and “low brow” movies
  - “High brow” movies: Schindler’s List, Like Water for Chocolate
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- Results:
  - Treatment 1 (immediate): 66% low brow
  - Treatment 2 (delay): 37% low brow