Econ 211

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The Standard Model: Exponential Discounting

Motivation

- Which would you rather have?
  - \$100 today OR \$95 one month
  - \$100 today OR \$97 one month
  - \$100 today OR \$99 one month
  - \$100 today OR \$101 one month
  - \$100 today OR \$103 one month
  - \$100 today OR \$105 one month
- If you value money today more than the same amount of money in the future, then we say you are *impatient*

Consumption Over Time

▶ Stream of consumption (or wealth or income) over T time periods, starting with period 1:

$$c = (c_1, c_2, c_3, \ldots, c_T)$$

▶ Example: $T = 3$ periods: $(c_1, c_2, c_3) = (\$5, \$10, \$0)$

▶ Utility is function of the entire stream of income:

$$U(c) = f(c_1, c_2, c_3, \ldots, c_T)$$

▶ If impatient, then would prefer to have an extra dollar today rather than tomorrow, implying

$$\frac{\partial U}{\partial c_t} > \frac{\partial U}{\partial c_{t+1}}$$

or equivalently:

$$\frac{\frac{\partial U}{\partial c_{t+1}}}{\frac{\partial U}{\partial c_t}} < 1$$

Exponential Discounting Model

- A popular model used in many fields of economics to represent impatient preferences
- Utility of consumption stream c:

$$U(c) = \sum_{t=1}^{T} \delta^{t-1} u(c_t)$$
$$= u(c_1) + \delta u(c_2) + \delta^2 u(c_3) + \ldots + \delta^{T-1} u(c_T)$$

- δ is called *discount factor*, where $0 < \delta < 1$
- Consumer will choose consumption stream that maximizes $U(c)$
- Is this utility function is impatient? Yes, since

$$\frac{\frac{\partial U}{\partial c_{t+1}}}{\frac{\partial U}{\partial c_t}} = \frac{\delta \frac{\partial u}{\partial c_{t+1}}}{\frac{\partial u}{\partial c_t}} = \delta < 1$$

- For δ closer to 0, agent becomes *more impatient*

Example: Doing Your Laundry

- Suppose your utility each day is proportional to how many clean outfits you have to wear
- On Friday that you have just 2 clean outfits left
- You can do laundry on Friday, Saturday, or Sunday, or Monday
- Doing laundry is annoying: -5 utils the day you choose to do it
- Doing laundry gets you 5 clean outfits, but you use one each day
- In summary:

	Utility on day			
	F	Sa	Su	M
Do laundry Fri	-3	5	4	3
Do laundry Sat	2	-4	5	4
Do laundry Sun	2	1	-5	5
Do laundry Mon	2	1	0	-5

When Do You Do Your Laundry?

▶ From Friday's perspective, what is overall utility of doing laundry on

Friday?	-3	$+\delta(5)$	$+\delta^2(4)$	$+\delta^3(3)$
Saturday?	2	$+\delta(-4)$	$+\delta^2(5)$	$+\delta^3(4)$
Sunday?	2	$+\delta(1)$	$+\delta^2(-5)$	$+\delta^3(5)$
Monday?	2	$+\delta(1)$	$+\delta^2(0)$	$+\delta^3(-5)$

▶ Utilities under various values of δ:

	Total utility if $\delta =$			
	1	0.6	0.52	0.25
Do laundry Fri	9*	2.09	1.10	-1.45
Do laundry Sat	7	2.27*	1.83	1.38
Do laundry Sun	3	1.88	1.87*	2.02
Do laundry Mon	-2	1.52	1.82	2.17*

Checking Follow Through

- Suppose your $\delta = 0.6$, so on Friday you decide to do laundry on Saturday
- Saturday morning comes, and you re-evaluate your choices
- Note that "today", ie period 1, is now Saturday
- From Saturday's perspective, what is utility of doing laundry on

Saturday?	-4	$+(0.6)(5)$	$+(0.6)^2(4)$	$= 0.44$
Sunday?	1	$+(0.6)(-5)$	$+(0.6)^2(5)$	$= -0.20$
Monday?	1	$+(0.6)(0)$	$+(0.6)^2(-5)$	$= -0.80$

- Will you follow through with plan? Yes, since utility of doing on Saturday is still highest

Time Consistency

- Consider decision maker planning consumption in future states
- If, when they arrive at the future state, they will not want to change their plan, then they are *time consistent*
- A formal definition
    - Let consumption for period τ chosen at period $t \leq \tau$ be $c(\tau|t)$
    - DM is *time consistent* if $c(\tau|t) = c(\tau|\tau)$ for any $t \leq \tau$
- Any exponential-discounting decision maker will be time-consistent

How Do We Measure Time Preferences?

- ▶ Suppose you are indifferent between \$100 today and \$X in one month
- ▶ Utility of \$100 today: $u(\$100)$
- ▶ Utility of \$X next month: $\delta u(\$X)$ (assuming monthly discount factor)
- ▶ Thus we must have $u(\$100) = \delta u(\$X)$, which implies

$$\delta = \frac{u(\$100)}{u(\$X)}$$

- ▶ If we make the assumption that $u(x) = x$, then

$$\delta = \frac{100}{X}$$

- ▶ Thus we can estimate time preferences by looking at switch point on price list

Evidence of Time Inconsistency

Time Inconsistency

- In actuality, we observe much behavior that is *time inconsistent*
    - That is, consumers make a different choice for tomorrow's consumption when asked today vs when asked tomorrow
    - Such consumers will have a *self-control problem*
- Also, we see that some people are aware of their time inconsistency
    - A *naive* agent believes (incorrectly) that he will follow through on his plans
    - A *sophisticated* agent knows that she may not follow through, so she may look for ways to *commit* herself to the plan

Lab Evidence: McClure et at (2007)

- Subjects told to come into the lab thirsty
- Experiment lasts at least 30 minutes
- Treatment 1 (immediate): choose either
    - 1 juice now (early) OR
    - 2 juices in 5 minutes (later)
- Treatment 2 (delay): choose either
    - 1 juice in 20 minutes (early) OR
    - 2 juices in 25 minutes (later)
- Subjects know this is their only chance to get a drink during the experiment

Results

- What behavior do we expect from discounted exponential utility model?
  - Break experiment up into five-minute periods
  - Treatment 1: choose early option if $u(1) > \delta u(2)$
  - Treatment 2: choose early option if $\delta^4 u(1) > \delta^5 u(2)$
    - Reduces to $u(1) > \delta u(2)$, same as Treatment 1
  - Thus we expect same percentage subjects choosing early option in both treatments
- What actually happened?
  - Treatment 1 (immediate): 60% choose early option
  - Treatment 2 (delay): 30% choose early option

Field Evidence: Read, Loewenstein, and Kalyanaraman (1999)

- ▶ Subjects get vouchers from certain movies off of a list
- ▶ List includes "high brow" and "low brow" movies
  - ▶ "High brow" movies: Schindler's List, Like Water for Chocolate
  - ▶ "Low brow" movies: The Mask, Mrs. Doubtfire
- ▶ Treatment 1 (immediate): Subjects pick movie for tonight
- ▶ Treatment 2 (delay): Subjects pick movie for one week from now
- ▶ Expect results from discounted exponential model?
  - ▶ As in previous experiment, expect same percentage choosing low brow movie in two treatments
- ▶ Results:
  - ▶ Treatment 1 (immediate): 66% low brow
  - ▶ Treatment 2 (delay): 37% low brow