Econ 211

Prof. Jeffrey Naecker

Wesleyan University

Motivation

- Last lecture, saw that
  - people appear relatively patient when all outcomes are in the future
  - people appear relatively impatient when trading off between right now and the future

Motivation

- Last lecture, saw that
  - people appear relatively patient when all outcomes are in the future
  - people appear relatively impatient when trading off between right now and the future
- Standard exponential discounting model does not allow for this
  - When trading off consumption between consecutive periods, discount factor is always δ
- We need a behavioral model which gives treats the present differently than all future periods

The $\beta - \delta$ Discounting Model

- First proposed by Strotz (1955) and popularized by Laibson (1997)
- Recall consumption stream $c = (c_1, c_2, \ldots, c_T)$
- Utility function:

$$\begin{aligned} U(c) &= u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3) + \ldots + \beta\delta^{T-1} u(c_T) \\ &= u(c_1) + \beta \left[\delta u(c_2) + \delta^2 u(c_3) + \ldots + \delta^{T-1} u(c_T)\right] \end{aligned}$$

  where
  - $0 \leq \beta \leq 1$
  - $0 \leq \delta \leq 1$
- Model known as β-δ *(beta-delta) discounting*

The $\beta - \delta$ Discounting Model

- First proposed by Strotz (1955) and popularized by Laibson (1997)
- Recall consumption stream $c = (c_1, c_2, \dots, c_T)$
- Utility function:

$$\begin{aligned} U(c) &= u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3) + \ldots + \beta\delta^{T-1} u(c_T) \\ &= u(c_1) + \beta \left[\delta u(c_2) + \delta^2 u(c_3) + \ldots + \delta^{T-1} u(c_T)\right] \end{aligned}$$

  where
  - $0 \leq \beta \leq 1$
  - $0 \leq \delta \leq 1$
- Model known as β-δ *(beta-delta) discounting*
- Embeds the standard exponential discounting model if $\beta = 1$
- All periods that are not the present are additionally discounted by β, so present period gets a relative boost
  - We say that this kind of preference has *present-bias*

Example: Present-Bias Leads to Time-Inconsistency

- Three periods: $t = 1, 2, 3$
- In period 2, choose what to eat
- Two options:
  1. Eat well: $u_2 = 5$, $u_3 = 10$
  2. Eat poorly: $u_2 = 8$, $u_3 = 6$
- Assume that DM has beta-delta preferences with $\beta = \frac{1}{2}$, $\delta = 1$

Example: Present-Bias Leads to Time-Inconsistency

- Three periods: $t = 1, 2, 3$
- In period 2, choose what to eat
- Two options:
    1. Eat well: $u_2 = 5$, $u_3 = 10$
    2. Eat poorly: $u_2 = 8$, $u_3 = 6$
- Assume that DM has beta-delta preferences with $\beta = \frac{1}{2}$, $\delta = 1$
- Decision in period 1?

Example: Present-Bias Leads to Time-Inconsistency

- Three periods: $t = 1, 2, 3$
- In period 2, choose what to eat
- Two options:
  1. Eat well: $u_2 = 5$, $u_3 = 10$
  2. Eat poorly: $u_2 = 8$, $u_3 = 6$
- Assume that DM has beta-delta preferences with $\beta = \frac{1}{2}$, $\delta = 1$
- Decision in period 1?
  - Eat well: $U = \frac{1}{2}5 + \frac{1}{2}10 = 7.5$
  - Eat poorly: $U = \frac{1}{2}8 + \frac{1}{2}6 = 7$
  - Decision: eat well

Example: Present-Bias Leads to Time-Inconsistency

- Three periods: $t = 1, 2, 3$
- In period 2, choose what to eat
- Two options:
  1. Eat well: $u_2 = 5$, $u_3 = 10$
  2. Eat poorly: $u_2 = 8$, $u_3 = 6$
- Assume that DM has beta-delta preferences with $\beta = \frac{1}{2}$, $\delta = 1$
- Decision in period 1?
  - Eat well: $U = \frac{1}{2}5 + \frac{1}{2}10 = 7.5$
  - Eat poorly: $U = \frac{1}{2}8 + \frac{1}{2}6 = 7$
  - Decision: eat well
- Decision in period 2:

Example: Present-Bias Leads to Time-Inconsistency

- Three periods: $t = 1, 2, 3$
- In period 2, choose what to eat
- Two options:
  1. Eat well: $u_2 = 5$, $u_3 = 10$
  2. Eat poorly: $u_2 = 8$, $u_3 = 6$
- Assume that DM has beta-delta preferences with $\beta = \frac{1}{2}$, $\delta = 1$
- Decision in period 1?
  - Eat well: $U = \frac{1}{2}5 + \frac{1}{2}10 = 7.5$
  - Eat poorly: $U = \frac{1}{2}8 + \frac{1}{2}6 = 7$
  - Decision: eat well
- Decision in period 2:
  - Eat well: $U = 5 + \frac{1}{2}10 = 10$
  - Eat poorly: $U = 8 + \frac{1}{1}6 = 11$
  - Decision: eat poorly

Procrastination

- In many economic problems, agent must do a task
  - Task needs to be done exactly once
  - Agent Has several time periods to do task
- To analyze these types of decisions, use *backwards induction*: start analysis at the end of the process and work back to the first period

Procrastination

- In many economic problems, agent must do a task
  - Task needs to be done exactly once
  - Agent Has several time periods to do task
- To analyze these types of decisions, use *backwards induction*: start analysis at the end of the process and work back to the first period
  - *Naive* agent is time inconsistent, but assumes self will be time-consistent in future
  - *Sophisticated* agent is time inconsistent, and knows self will be time-inconsistent in future

Example: Paper Writing

- Suppose student has a paper due in 4 weeks
- Can write the paper on weekend 1, 2, 3, or 4
- Cost of writing paper is missing going to movies with friends:
  - Weekend 1: bad movie, cost $= 3$
  - Weekend 2: OK movie, cost $= 5$
  - Weekend 3: good movie, cost $= 8$
  - Weekend 4: great movie, cost $= 13$
- Benefit of writing the paper is $\bar{v} > 0$, received in week 5 when grades are given
- For all types of agents, assume $\delta = 1$ in what follows
- For time-inconsistent types, assume $\beta = \frac{1}{2}$

When Does Time-Consistent Agent Write Paper?

- Decision in week 4:

When Does Time-Consistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made

When Does Time-Consistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:

When Does Time-Consistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\bar{v} - 8$
  - If wait till next week, utility is $\bar{v} - 13$
  - So, will do paper in week 3 (if not done already)

When Does Time-Consistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\bar{v} - 8$
  - If wait till next week, utility is $\bar{v} - 13$
  - So, will do paper in week 3 (if not done already)
- Decision in week 2:

When Does Time-Consistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\bar{v} - 8$
  - If wait till next week, utility is $\bar{v} - 13$
  - So, will do paper in week 3 (if not done already)
- Decision in week 2:
  - If do paper, utility is $\bar{v} - 5$
  - If don't write paper, know will write next week for utility $\bar{v} - 8$
  - So, will do paper in week 2 (if not done already)

When Does Time-Consistent Agent Write Paper?

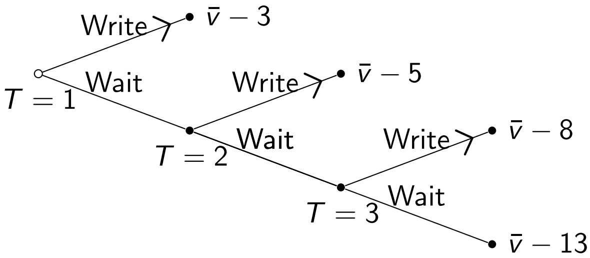
- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\bar{v} - 8$
  - If wait till next week, utility is $\bar{v} - 13$
  - So, will do paper in week 3 (if not done already)
- Decision in week 2:
  - If do paper, utility is $\bar{v} - 5$
  - If don't write paper, know will write next week for utility $\bar{v} - 8$
  - So, will do paper in week 2 (if not done already)
- Decision in week 1:

When Does Time-Consistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\bar{v} - 8$
  - If wait till next week, utility is $\bar{v} - 13$
  - So, will do paper in week 3 (if not done already)
- Decision in week 2:
  - If do paper, utility is $\bar{v} - 5$
  - If don't write paper, know will write next week for utility $\bar{v} - 8$
  - So, will do paper in week 2 (if not done already)
- Decision in week 1:
  - If do paper, utility is $\bar{v} - 3$
  - If don't write paper, know will write next week for utility $\bar{v} - 5$
  - So, will do paper in week 1

Decision Tree

- Helpful to keep track of decisions of agent with a decision tree
- Note decision each period is *whether* to write or wait, not *when* to write



When Does Naive Time-Inconsistent Agent Write Paper?

- Decision in week 4:

When Does Naive Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made

When Does Naive Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:

When Does Naive Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 8$
  - If wait till next week, utility is $\frac{1}{2}\bar{v} - \frac{1}{2}13 = \frac{1}{2}\bar{v} - 6.5$
  - So, will choose NOT to do paper in week 3

When Does Naive Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 8$
  - If wait till next week, utility is $\frac{1}{2}\bar{v} - \frac{1}{2}13 = \frac{1}{2}\bar{v} - 6.5$
  - So, will choose NOT to do paper in week 3
- Decision in week 2:

When Does Naive Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 8$
  - If wait till next week, utility is $\frac{1}{2}\bar{v} - \frac{1}{2}13 = \frac{1}{2}\bar{v} - 6.5$
  - So, will choose NOT to do paper in week 3
- Decision in week 2:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 5$
  - If don't write paper:
    - Remember, thinks future self is time-consistent
    - So, thinks (incorrectly!) that will do paper in week 3
    - From perspective of week 2, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}8 = \frac{1}{2}\bar{v} - 4$
  - So, will choose NOT to do paper in week 2

Naive Time-Inconsistent Agent, con't

- Decision in week 1:

Naive Time-Inconsistent Agent, con't

- Decision in week 1:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 3$
  - If don't write paper:
    - Remember, thinks future self is time-consistent
    - So, thinks (incorrectly!) that will do paper in week 2
    - From perspective of week 1, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}5 = \frac{1}{2}\bar{v} - 2.5$
  - So, will choose NOT to do paper in week 1

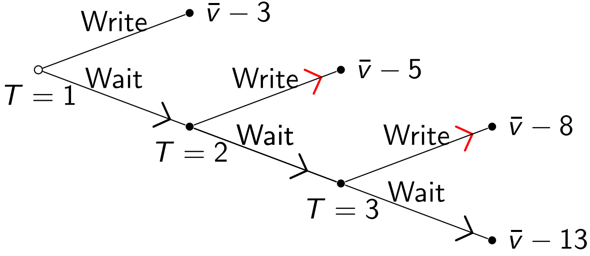
Naive Time-Inconsistent Agent, con't

- Decision in week 1:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 3$
  - If don't write paper:
    - Remember, thinks future self is time-consistent
    - So, thinks (incorrectly!) that will do paper in week 2
    - From perspective of week 1, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}5 = \frac{1}{2}\bar{v} - 2.5$
  - So, will choose NOT to do paper in week 1
- Overall result:

Naive Time-Inconsistent Agent, con't

- Decision in week 1:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 3$
  - If don't write paper:
    - Remember, thinks future self is time-consistent
    - So, thinks (incorrectly!) that will do paper in week 2
    - From perspective of week 1, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}5 = \frac{1}{2}\bar{v} - 2.5$
  - So, will choose NOT to do paper in week 1
- Overall result:
  - Agent waits until week 4 to do paper, misses best movie

Decision Tree for Naive Agent



When Does Sophisticated Time-Inconsistent Agent Write Paper?

- Decision in week 4:

When Does Sophisticated Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made

When Does Sophisticated Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:

When Does Sophisticated Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 8$
  - If wait till next week, utility is $\frac{1}{2}\bar{v} - \frac{1}{2}13 = \frac{1}{2}\bar{v} - 6.5$
  - So, will choose NOT to do paper in week 3

When Does Sophisticated Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 8$
  - If wait till next week, utility is $\frac{1}{2}\bar{v} - \frac{1}{2}13 = \frac{1}{2}\bar{v} - 6.5$
  - So, will choose NOT to do paper in week 3
- Decision in week 2:

When Does Sophisticated Time-Inconsistent Agent Write Paper?

- Decision in week 4:
  - Have to do the paper at this point, so no real choice to be made
- Decision in week 3:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 8$
  - If wait till next week, utility is $\frac{1}{2}\bar{v} - \frac{1}{2}13 = \frac{1}{2}\bar{v} - 6.5$
  - So, will choose NOT to do paper in week 3
- Decision in week 2:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 5$
  - If don't write paper:
    - Remember, knows future self is time-inconsistent
    - So, thinks (correctly) that will NOT do paper in week 3
    - From perspective of week 2, utility of waiting is $\frac{1}{2}\bar{v} - 6.5$
  - So, will choose to DO paper in week 2

Sophisticated Time-Inconsistent Agent, con't

- Decision in week 1:

Sophisticated Time-Inconsistent Agent, con't

- Decision in week 1:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 3$
  - If don't write paper:
    - Remember, knows future self is time-inconsistent
    - So, thinks (correct) that will do paper in week 2
    - From perspective of week 1, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}5 = \frac{1}{2}\bar{v} - 2.5$
  - So, will choose NOT to do paper in week 1

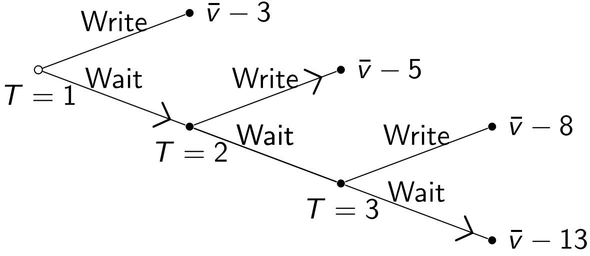
Sophisticated Time-Inconsistent Agent, con't

- Decision in week 1:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 3$
  - If don't write paper:
    - Remember, knows future self is time-inconsistent
    - So, thinks (correct) that will do paper in week 2
    - From perspective of week 1, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}5 = \frac{1}{2}\bar{v} - 2.5$
  - So, will choose NOT to do paper in week 1
- Overall result:

Sophisticated Time-Inconsistent Agent, con't

- Decision in week 1:
  - If do paper, utility is $\frac{1}{2}\bar{v} - 3$
  - If don't write paper:
    - Remember, knows future self is time-inconsistent
    - So, thinks (correct) that will do paper in week 2
    - From perspective of week 1, utility of waiting is $\frac{1}{2}\bar{v} - \frac{1}{2}5 = \frac{1}{2}\bar{v} - 2.5$
  - So, will choose NOT to do paper in week 1
- Overall result:
  - Agent waits until week 2 to do paper, meaning agent misses OK movie

Decision Tree for Sophisticated Agent



Measuring $\beta - \delta$ Preferences

- Again, assume for simplicity that $u(x) = x$
- Suppose you say that you are indifferent between \$100 in one month and \$Y in two months
- Then we must have $\beta\delta 100 = \beta\delta^2 Y$, which implies

$$\delta = \frac{100}{Y}$$

- Suppose additionally you are indifferent between \$100 today and \$X in one month
- Then we must have $100 = \beta\delta X$
- Together with the equation for δ above, this implies

$$\beta = \frac{Y}{X}$$

More Details on Multiple Price List Methodology

- Most commonly used experimental method to estimate individual time preferences
- Choices between a smaller, sooner reward and a later, larger reward
- Typically one option stays fixed will the other varies
- Point at which subject switches from smaller/sooner reward to larger/later reward helps estimate their time preference parameters
- Essential to have both delay $= 0$ and delay > 0 list to separately identify β and δ

Wrap-up

- No required reading for this lecture
- However, post in discussion forum your idea for an original research study
  - Can be experiment (lab or field) or observational study
  - Must be more than straight replication of existing study we've seen so far in class
  - Describe design, subject population, and expected results
  - Please also give feedback on at least one classmate's idea