Econ 211

Prof. Jeffrey Naecker

Wesleyan University

Failure of Standard Solution Concepts in Beauty Contest Game

- Clearly Nash equilibrium does not hold
- Many players even choose dominated strategies
- Yet clearly subjects are not playing (completely) randomly

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- What is the limit of this process? Note that we can keep doing this until we hit 0

New Solution Concept: Level k

- Define recursive set of strategies:
  - Level 0: Naive or non-strategic play
    - Guessing randomly in guessing game, but other assumptions make sense in other games
  - Level 1: best-respond to level 0
  - Level 2: best-respond to level 1 ...
  - Level k: best-respond to level $k-1$
- New solution concept: players will select one of the level k strategies, typically for $k = 1, 2$, or 3
- Note that Level k converges to NE as $k \to \infty$

Level k Predictions in Guessing Games

- Note that level-k predicts we should see behavior cluster at 50, $50p$, $50p^2$, and so on
- Examining the data from Nagel (1995), we see
  - $p = \frac{1}{2}$: clusters at 25, 12.5
  - $p = \frac{2}{3}$: clusters at 33, 22
    - Also note cluster at 67: what is going on here?
  - $p = \frac{4}{3}$: clusters at 67, 88

Another Kind of Guessing Game

- Suppose you are playing with a partner
- You and your partner both submit guesses between 1 and 19
- Your payoffs:
  - Always get your guess in dollars
  - If your guess is exactly 3 less than opponent's guess, you get an additional bonus of 50 dollars
  - If your guess is exactly equal to opponent's guess, you get an additional bonus of 25 dollars

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- What is/are NE?

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- $L2$?
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- What is/are NE?
  - Level k converges towards guessing 1
  - If both guess 1, neither player can profitably deviate
    - Getting payoff $1 + 25 = 26$
    - Raising guess to x will get payoff $x \leq 19$

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  - Same for 2 and 3
  - Thus NE are $(1, 1)$, $(2, 2)$, and $(3, 3)$
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  - Guessing 17, 18 or 19 dominated
  - No dominant strategies

Summary

- Solution concepts make predictions about what strategies will be played:
  - Nash Equilibrium: mutual best response
  - Dominant strategy: always a best response
  - Dominated strategy: never a best response
  - Level k: iterative best responses
- In experiments, we see that people
  - do not always play Nash
  - sometimes choose dominated strategies
  - often play $L1$, $L2$, or $L3$