

Econ 211

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What is a Game?

- ▶ First, we have several building blocks:
 - ▶ A *player* is a decision-maker in the game
 - ▶ A *strategy* is a complete contingent plan that a player makes for every possible point in the game where she can make a decision
 - ▶ A *payoff function* tells us what the utility of each player will be as a function of all their strategies
- ▶ A game (in normal form) is a set of players, a set of possible strategies for those players, and a payoff function

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Motivation

- ▶ We need a tool for analyzing behavior when we have more than one decision-maker
- ▶ In many cases, we can assume competitive markets with large numbers of decision-makers
 - ▶ No one agent has a noticeable impact on the outcome
- ▶ However, we often end up in situations where the typical market assumptions do not hold
- ▶ This is where game theory becomes useful

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Example of a Game

- ▶ We can represent a normal-form game with a matrix
 - ▶ Rows indicate strategies for player 1
 - ▶ Columns indicate strategies for player 2
 - ▶ Cells show payoffs for the two players
 - ▶ Usually put player 1 (row player) payoffs first in list
- ▶ For example, a famous game called the Prisoner's Dilemma
 - ▶ Players can either cooperate (C) or defect (D)
- ▶ Payoff matrix:

	C	D
C	(-2, -2)	(-5, -1)
D	(-1, -5)	(-4, -4)

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Solution Concepts

- ▶ A *solution concept* is a rule that, given any game, predicts which outcome(s) will actually happen when people play the game
- ▶ Focus on three solution concepts from classic game theory:
 - ▶ Nash Equilibrium
 - ▶ Dominant Strategies
 - ▶ Dominated Strategies

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Nash Equilibrium of Prisoner's Dilemma

	C	D
C	(-2, -2)	(-5, -1)
D	(-1, -5)	(-4, -4)

- ▶ What is the Nash equilibrium of the Prisoner's Dilemma?

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Nash Equilibrium

- ▶ Strategies for row player: r_1, r_2, r_3, \dots
- ▶ Strategies for column player: c_1, c_2, c_3, \dots
- ▶ Let $BR_r(c)$ be the row player's best response function
 - ▶ That is, if column player is playing c , row player can maximize payoff by playing $BR_r(c)$
- ▶ Similarly, let $BR_c(r)$ be the column player's best response function

Definition

The strategies r^{NE}, c^{NE} are a *Nash Equilibrium* if

$$r^{NE} = BR_r(c^{NE}) \quad \text{and} \quad c^{NE} = BR_c(r^{NE}).$$

- ▶ That is, both players are best-responding to each other
- ▶ Check NE by ensuring that no player has incentive to deviate

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Dominant Strategies

- ▶ The strategy r^D is a *dominant strategy* iff

$$r^D = BR_r(c) \quad \text{for all } c = c_1, c_2, c_3, \dots$$

- ▶ That is, r^D is *always* the row player's best response, regardless of what the column player is doing
- ▶ Definition is similar for column player
- ▶ If both players have a dominant strategy, then the game has a *dominant strategy solution*

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Dominated Strategies

- ▶ A strategy is *dominated* if it is *never* the best response for a player
- ▶ This gives us another solution concept: players will not play dominated strategies
- ▶ Relation to dominant strategies:
 - ▶ Possible to have strategies that are neither dominant nor dominated
 - ▶ In simple 2-by-2 games: if one strategy is dominant, other will be dominated
 - ▶ In more complex games: possible to have strategies that are dominated even if there is not dominant strategy

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Prisoner's Dilemma

	<i>C</i>	<i>D</i>
<i>C</i>	(-2, -2)	(-5, -1)
<i>D</i>	(-1, -5)	(-4, -4)

- ▶ Does the Prisoner's dilemma have any dominant or dominated strategies?

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Common Knowledge of Rationality

- ▶ Note that Nash Equilibrium has a key assumption built in
 - ▶ Players must assume that all other players are capable of calculating their best response
 - ▶ Must assume that all other players know that they know this
 - ▶ And that all player know that they know that they know this
 - ▶ And so on . . .
 - ▶ This is called *common knowledge of rationality*
- ▶ Dominant/dominated strategies assume less about the other players, so don't need common knowledge
 - ▶ But as a result, dominant/dominated strategies will in general make less specific predictions about outcomes of a game
 - ▶ Dominant/dominated strategies may not exist at all, in fact, in which case that concept makes no prediction at all
- ▶ Is the assumption of common knowledge of rationality a good assumption for human behavior?

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Sequential Games

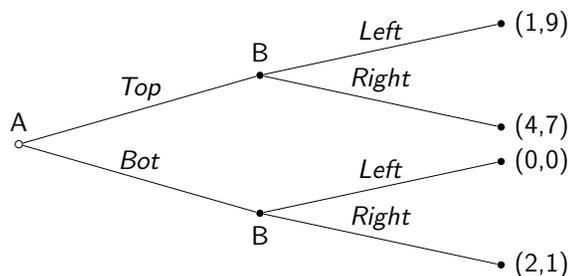
- ▶ Consider the following game
 - ▶ Player A chooses Top or Bottom
 - ▶ Observing A's choice, player B then chooses Left or Right
- ▶ This is a *sequential game*, because players move in sequence rather than simultaneously
- ▶ Payoff function:

(Top, Left)	→ (1, 9)
(Top, Right)	→ (4, 7)
(Bottom, Left)	→ (0, 0)
(Bottom, Right)	→ (2, 1)
- ▶ Note Player B really now has more complicated strategies, since must pick what to do after each move player B

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Extensive Form

- ▶ We analyze such games in *extensive form* with a game tree:



- ▶ Note that extensive form has:
 - ▶ Every non-terminal node labeled with player who moves at that point
 - ▶ Every terminal node labeled with payoffs
 - ▶ Every branch labeled with available actions

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Solution Concept: Subgame Perfect Nash Equilibrium

- ▶ We solve extensive form games with *backwards induction*
 - ▶ Start with end of the game tree
 - ▶ Determine what last mover will do
 - ▶ Take one step backwards in tree and repeat until all decisions have been analyzed
- ▶ The solution we arrive at is called the *subgame perfect Nash equilibrium*
- ▶ Note that in sequential games, strategies must list action at every node at which the player moves
 - ▶ For example, player B's strategy must indicate what B will do if A plays Top *and* what B will do if A plays Bot
 - ▶ Notation: Right/Left means play Right if Top, Left if Bottom, for example

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Example

- ▶ What is backwards induction solution to game on previous slide?

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Application: Ultimatum Game

- ▶ Consider ultimatum game with pie of size one
- ▶ Suppose proposer is selfish but responder has Fehr-Schmidt preferences with $\alpha = \beta = \frac{1}{2}$
- ▶ What is SPNE outcome of this game?

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How do People Actually Play Games?

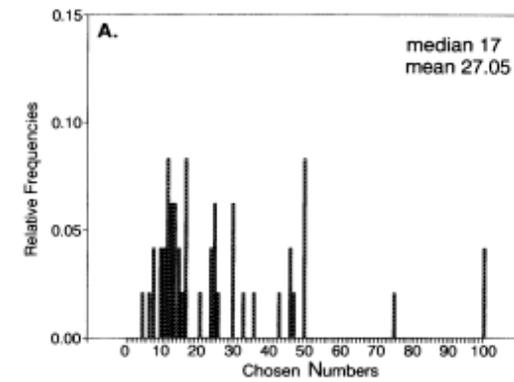
- ▶ Nagel (1995) examines *beauty contest game*, also known as *guessing game*
 - ▶ Large number of players M
 - ▶ Positive number p is told to players (assume $2p \leq M$)
 - ▶ Each player picks a number from 0 to 100
 - ▶ Average guess X is calculated
 - ▶ Player closest to pX wins a prize
- ▶ What are the NE of this game?

- ▶ What are the dominated strategies in this game?

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How Do People Actually Play?

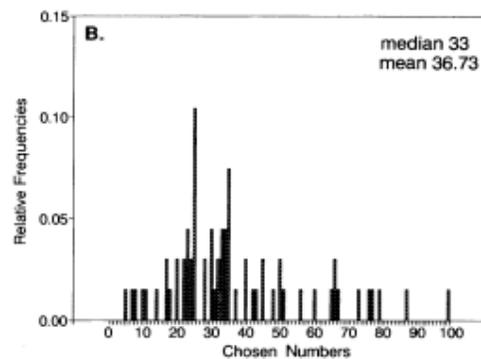
- ▶ $p = \frac{1}{2}$



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How Do People Actually Play?

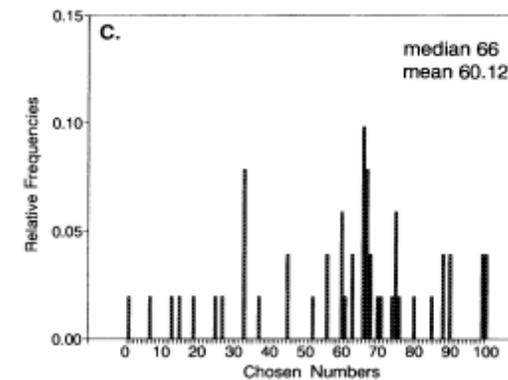
- ▶ $p = \frac{2}{3}$



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How Do People Actually Play?

- ▶ $p = \frac{4}{3}$



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Failure of Standard Solution Concepts

- ▶ Clearly Nash equilibrium does not hold
- ▶ Many players even choose dominated strategies
- ▶ Yet clearly subjects are not playing (completely) randomly
- ▶ So, need behavioral game theory