Econ 211

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Motivation

- We need a tool for analyzing behavior when we have more than one decision-maker
- In many cases, we can assume competitive markets with large numbers of decision-makers
  - No one agent has a noticeable impact on the outcome
- However, we often end up in situations where the typical market assumptions do not hold
- This is where game theory becomes useful

What is a Game?

- First, we have several building blocks:
    - A *player* is a decision-maker in the game
    - A *strategy* is a complete contingent plan that a player makes for every possible point in the game where she can make a decision
    - A *payoff function* tells us what the utility of each player will be as a function of all their strategies
- A game (in normal form) is a set of players, a set of possible strategies for those players, and a payoff function

Example of a Game

- We can represent a normal-form game with a matrix
  - Rows indicate strategies for player 1
  - Columns indicate strategies for player 2
  - Cells show payoffs for the two players
    - Usually put player 1 (row player) payoffs first in list

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- For example, a famous game called the Prisoner's Dilemma
  - Players can either cooperate (C) or defect (D)
- Payoff matrix:

	C	D
C	$(-2,-2)$	$(-5,-1)$
D	$(-1,-5)$	$(-4,-4)$

Solution Concepts

- A *solution concept* is a rule that, given any game, predicts which outcome(s) will actually happen when people play the game
- Focus on three solution concepts from classic game theory:
  - Nash Equilibrium
  - Dominant Strategies
  - Dominated Strategies

Nash Equilibrium

- Strategies for row player: $r_1, r_2, r_3, \ldots$
- Strategies for column player: $c_1, c_2, c_3, \ldots$
- Let $BR_r(c)$ be the row player's best response function
  - That is, if column player is playing c, row player can maximize payoff by playing $BR_s(c)$
- Similarly, let $BR_c(r)$ be the column player's best response function

Definition

The strategies r^{NE}, c^{NE} are a *Nash Equilibrium* if

$$r^{NE} = BR_r(c^{NE}) \quad \text{and} \quad c^{NE} = BR_c(r^{NE}).$$

- That is, both players are best-responding to each other
- Check NE by ensuring that no player has incentive to deviate

Nash Equilibrium of Prisoner's Dilemma

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- What is the Nash equilibrium of the Prisoner's Dilemma?
  - If your opponent is choosing cooperate, your best response to choose defect, since $(-1 > -2)$
  - If your opponent is choosing defect, your best response to choose defect, since $(-4 > -5)$
  - Thus NE is that both players defect

Dominant Strategies

- The strategy r^D is a *dominant strategy* iff

$$r^D = BR_r(c) \quad \text{for all } c = c_1, c_2, c_3, \ldots.$$

- That is, r^D is *always* the row player's best response, regardless of what the column player is doing
- Definition is similar for column player
- If both players have a dominant strategy, then the game has a *dominant strategy solution*

Dominated Strategies

- A strategy is *dominated* if it is *never* the best response for a player
- This gives us another solution concept: players will not play dominated strategies
- Relation to dominant strategies:
  - Possible to have strategies that are neither dominant nor dominated
  - In simple 2-by-2 games: if one strategy is dominant, other will be dominated
  - In more complex games: possible to have strategies that are dominated even if there is not dominant strategy

Prisoner's Dilemma

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- Does the Prisoner's dilemma have any dominant or dominated strategies?
  - Defect is a dominant strategy for both players
  - Cooperate is a dominated strategy for both players
  - Thus the only possible outcome is (Defect, Defect)

Common Knowledge of Rationality

- Note that Nash Equilibrium has a key assumption built in
  - Players must assume that all other players are capable of calculating their best response
  - Must assume that all other players know that they know this
  - And that all player know that they know that they know this
  - And so on ...

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- Dominant/dominated strategies assume less about the other players, so don't need common knowledge
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- Is the assumption of common knowledge of rationality a good assumption for human behavior?

Sequential Games

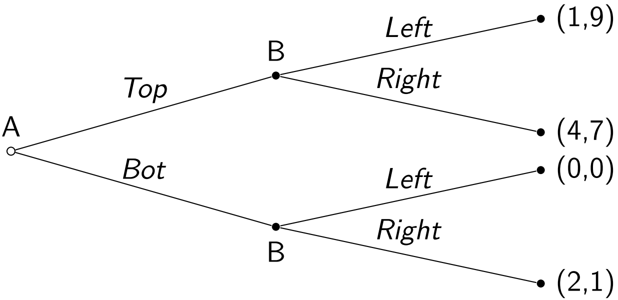
- Consider the following game
  - Player A chooses Top or Bottom
  - Observing A's choice, player B then chooses Left or Right
- This is a *sequential game*, because players move in sequence rather than simultaneously
- Payoff function:

$$
\begin{array}{ll}
\text{(Top, Left)} & \rightarrow (1, 9) \\
\text{(Top, Right)} & \rightarrow (4, 7) \\
\text{(Bottom, Left)} & \rightarrow (0, 0) \\
\text{(Bottom, Right)} & \rightarrow (2, 1)
\end{array}
$$

- Note Player B really now has more complicated strategies, since must pick what to do after each move player B

Extensive Form

- We analyze such games in *extensive form* with a game tree:



- Note that extensive form has:
  - Every non-terminal node labeled with player who moves at that point
  - Every terminal node labeled with payoffs
  - Every branch labeled with available actions

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- We solve extensive form games with *backwards induction*
  - Start with end of the game tree
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- The solution we arrive at is called the *subgame perfect Nash equilibrium*
- Note that in sequential games, strategies must list action at every node at which the player moves
  - For example, player B's strategy must indicate what B will do if A plays Top *and* what B will do if A plays Top
  - Notation: Right/Left means play Right if Top, Left if Bottom, for example

Example

- What is backwards induction solution to game on previous slide?
  - After Top, player B will play Left
  - After Bottom, player B will play Right
  - Given what player B will do, player A will choose to play Bottom
  - SPNE <u>strategies</u> are (Bottom, Left/Right)
  - SPNE <u>outcome</u> is (Bottom, Right)

Application: Ultimatum Game

- Consider ultimatum game with pie of size one
- Suppose proposer is selfish but responder has Fehr-Schmidt preferences with $\alpha = \beta = \frac{1}{2}$
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  - Proposer chooses $x = \frac{1}{4}$ (which is accepted)

How do People Actually Play Games?

- Nagel (1995) examines *beauty contest game*, also known as *guessing game*
  - Large number of players M
  - Positive number p is told to players (assume $2p \leq M$)
  - Each player picks a number from 0 to 100
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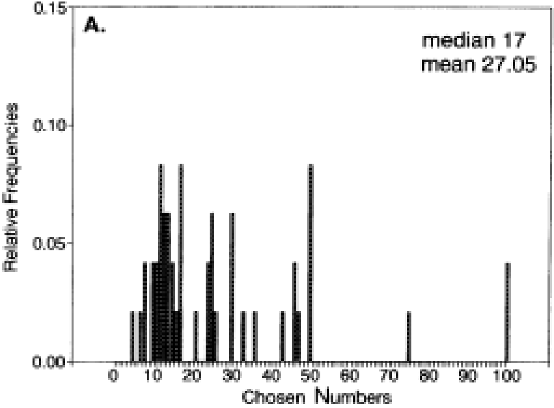
- What are the dominated strategies in this game?
  - If $p < 1$, any guess above $100p$ is dominated

How Do People Actually Play?

- $p = \frac{1}{2}$



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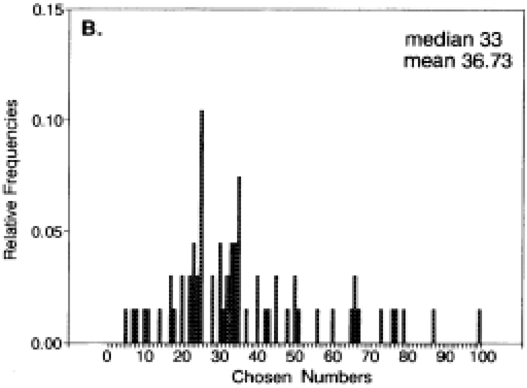
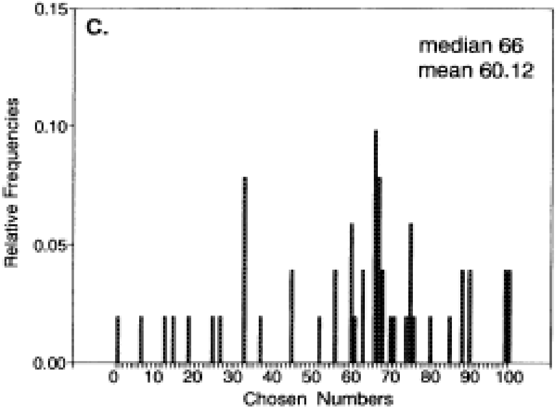


How Do People Actually Play?

- $p = \frac{4}{3}$



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- Many players even choose dominated strategies
- Yet clearly subjects are not playing (completely) randomly
- So, need behavioral game theory