Econ 301: Microeconomic Analysis

Prof. Jeffrey Naecker

Wesleyan University

Cost Functions

Cost Functions

- So far we have written the cost function $c(w_1, w_2, y)$
- From now on, we will use shorthand $c(y)$

Cost Functions

- So far we have written the cost function $c(w_1, w_2, y)$
- From now on, we will use shorthand $c(y)$
- We will break the cost function into two parts:

$$c(y) = c_v(y) + F$$

  - $c_v(y)$ is the *variable cost*, also written $VC(y)$
  - F is the *fixed cost*

Cost Functions

- So far we have written the cost function $c(w_1, w_2, y)$
- From now on, we will use shorthand $c(y)$
- We will break the cost function into two parts:

$$c(y) = c_v(y) + F$$

  - $c_v(y)$ is the *variable cost*, also written $VC(y)$
  - F is the *fixed cost*
- We can divide the cost function by the output to get the *average cost* function:

$$AC(y) = \frac{c(y)}{y} = \frac{c_v(y)}{y} + \frac{F}{y}$$

  - $\frac{c_v(y)}{y}$ is the *average variable cost*, $AVC(y)$
  - $\frac{F}{y}$ is the *average fixed cost*, $AFC(y)$

Average Cost Graphically

- How does AFC depend on y?

Average Cost Graphically

- How does AFC depend on y? Decreasing in y

Average Cost Graphically

- How does AFC depend on y? Decreasing in y
- How does AVC depend on y?

Average Cost Graphically

- How does AFC depend on y? Decreasing in y
- How does AVC depend on y?
  - Technology may start out at increasing returns to scale
  - Eventually returns to scale will become decreasing
  - Thus AVC may start out decreasing but eventually increasing in y

Average Cost Graphically

- How does AFC depend on y? Decreasing in y
- How does AVC depend on y?
  - Technology may start out at increasing returns to scale
  - Eventually returns to scale will become decreasing
  - Thus AVC may start out decreasing but eventually increasing in y
- Adding these up, we see that AC is initially decreasing, then increasing, in y

Average Cost Graphically

- How does AFC depend on y? Decreasing in y
- How does AVC depend on y?
  - Technology may start out at increasing returns to scale
  - Eventually returns to scale will become decreasing
  - Thus AVC may start out decreasing but eventually increasing in y
- Adding these up, we see that AC is initially decreasing, then increasing, in y

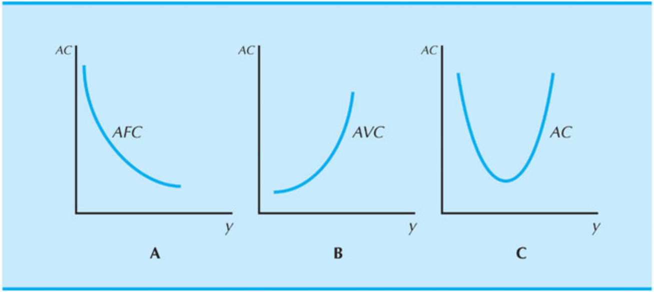


Figure 22.1

Marginal Cost

- Useful to define the *marginal cost* curve:

$$MC(y) = c'(y) = c_v'(y)$$

- MC is the slope of the cost function
- Measures how the cost of producing an additional unit of output changes as the total output level changes

MC and AVC Have Same Vertical Intercepts

- MC curve and AVC curve approach the same height as y approaches 0

MC and AVC Have Same Vertical Intercepts

- MC curve and AVC curve approach the same height as y approaches 0
- Formal proof:
  - The limit of the AVC curve as y approaches 0 is

$$\lim_{y \to 0} \frac{c_v(y)}{y}$$

  - Note we can't just plug in 0, since numerator and denominator both go to zero
  - But using l'Hopital's rule, we can find the limit:

$$\lim_{y \to 0} AVC(y) = \lim_{y \to 0} \frac{c_v(y)}{y} = \frac{\lim_{y \to 0} c_v'(y)}{\lim_{y \to 0} 1} = \lim_{y \to 0} c_v'(y) = \lim_{y \to 0} MC(y)$$

MC Intersects AVC at Minimum

- MC curve intersects the AVC curve where the AVC curve is minimized

MC Intersects AVC at Minimum

- MC curve intersects the AVC curve where the AVC curve is minimized
- Consider range where AVC is decreasing
  - As output increases, must be adding a number smaller than the average to the total
  - Thus $MC(y) < AVC(y)$ in this range

MC Intersects AVC at Minimum

- MC curve intersects the AVC curve where the AVC curve is minimized
- Consider range where AVC is decreasing
  - As output increases, must be adding a number smaller than the average to the total
  - Thus $MC(y) < AVC(y)$ in this range
- Consider range where AVC is increasing
  - As output increases, must be adding a number larger that the average to the total
  - Thus $MC(y) > AVC(y)$ in this range

MC Intersects AVC at Minimum

- MC curve intersects the AVC curve where the AVC curve is minimized
- Consider range where AVC is decreasing
  - As output increases, must be adding a number smaller than the average to the total
  - Thus $MC(y) < AVC(y)$ in this range
- Consider range where AVC is increasing
  - As output increases, must be adding a number larger that the average to the total
  - Thus $MC(y) > AVC(y)$ in this range
- Thus $MC(y) = AVC(y)$ at the point where $AVC(y)$ is neither increasing nor decreasing, ie the minimum

MC Intersects AC at Minimum

- The argument on the previous page applies to the average cost curve as well
- Thus we have $MC(y) = AC(y)$ at the point where $AC(y)$ is minimized
- Note because *AC* curve lies above *AVC* curve, minimum of AC curve is to the right of minimum of *AVC* curve

Marginal Cost Graphically

- These three properties allow us to draw the MC curve given the AVC and AC curves:
    1. Marginal cost starts out at the same level as average variable cost
        - Note if AVC is initially decreasing, MC will initially decrease as well

Marginal Cost Graphically

- These three properties allow us to draw the MC curve given the AVC and AC curves:
  1. Marginal cost starts out at the same level as average variable cost
     - Note if AVC is initially decreasing, MC will initially decrease as well
  2. Marginal cost intersects AVC curve at AVC minimum

Marginal Cost Graphically

- These three properties allow us to draw the MC curve given the AVC and AC curves:
  1. Marginal cost starts out at the same level as average variable cost
     - Note if AVC is initially decreasing, MC will initially decrease as well
  2. Marginal cost intersects AVC curve at AVC minimum
  3. Marginal cost intersects AC curve at AC minimum

Marginal Cost Graphically

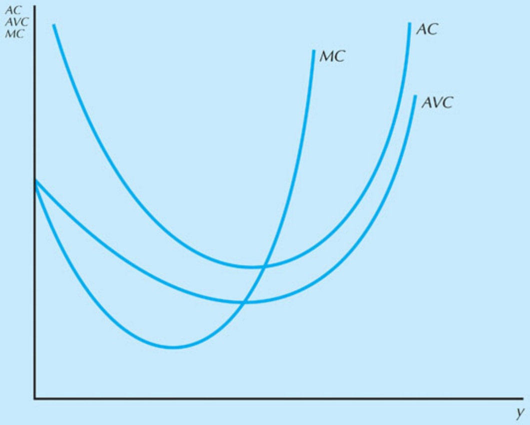


Figure 22.2

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost?

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost?

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost? $AVC(y) = y$

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost? $AVC(y) = y$
- What is fixed cost?

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost? $AVC(y) = y$
- What is fixed cost? $FC(y) = 1$

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost? $AVC(y) = y$
- What is fixed cost? $FC(y) = 1$
- What is average fixed cost?

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost? $AVC(y) = y$
- What is fixed cost? $FC(y) = 1$
- What is average fixed cost? $AFC(y) = \frac{1}{y}$

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost? $AVC(y) = y$
- What is fixed cost? $FC(y) = 1$
- What is average fixed cost? $AFC(y) = \frac{1}{y}$
- What is average cost?

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost? $AVC(y) = y$
- What is fixed cost? $FC(y) = 1$
- What is average fixed cost? $AFC(y) = \frac{1}{y}$
- What is average cost? $AC(y) = y + \frac{1}{y}$

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost? $AVC(y) = y$
- What is fixed cost? $FC(y) = 1$
- What is average fixed cost? $AFC(y) = \frac{1}{y}$
- What is average cost? $AC(y) = y + \frac{1}{y}$
- What is marginal cost?

Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost? $VC(y) = y^2$
- What is average variable cost? $AVC(y) = y$
- What is fixed cost? $FC(y) = 1$
- What is average fixed cost? $AFC(y) = \frac{1}{y}$
- What is average cost? $AC(y) = y + \frac{1}{y}$
- What is marginal cost? $MC(y) = 2y$

Cost Curve Example, con't

- Where is minimum of *AVC* curve?

Cost Curve Example, con't

- Where is minimum of AVC curve? $AVC = 0$ at $y = 0$

Cost Curve Example, con't

- Where is minimum of AVC curve? $AVC = 0$ at $y = 0$
- Where is minimum of AC curve?

Cost Curve Example, con't

- Where is minimum of AVC curve? $AVC = 0$ at $y = 0$
- Where is minimum of AC curve?
  - $\min_y y + \frac{1}{y}$ implies FOC $1 - \frac{1}{y^2} = 0$
  - Solving gives $y = 1$
  - $AC(1) = 2$

Cost Curve Example, con't

- Where is minimum of AVC curve? $AVC = 0$ at $y = 0$
- Where is minimum of AC curve?
  - $\min_y y + \frac{1}{y}$ implies FOC $1 - \frac{1}{y^2} = 0$
  - Solving gives $y = 1$
  - $AC(1) = 2$
- Does the MC curve go through these points?

Cost Curve Example, con't

- Where is minimum of *AVC* curve? $AVC = 0$ at $y = 0$
- Where is minimum of *AC* curve?
  - $\min_y y + \frac{1}{y}$ implies FOC $1 - \frac{1}{y^2} = 0$
  - Solving gives $y = 1$
  - $AC(1) = 2$
- Does the *MC* curve go through these points? Yes:
  - $MC(0) = 0$
  - $MC(1) = 2$

Firm Supply

Firm Constraints

- How much output the firm sells depends on two constraints
  1. Technology constraints
  2. Market constraints
- We have focused so far on technology
- It is now time to consider the role of the market
  - Many possible assumptions for what type of market we are in, eg monopoly, oligopoly
  - For now we will assume we are in a perfectly competitive market, ie firms are price takers

Demand Curve Faced by the Firm

- We call the relationship between the price the firm sets and the amount it sells at that price the *demand curve faced by the firm*
- This is not the same as the market demand curve in general
  - Though that is true for monopoly
- What is the demand curve faced by the firm for perfect competition, assuming market price p^*?

Demand Curve Faced by the Firm

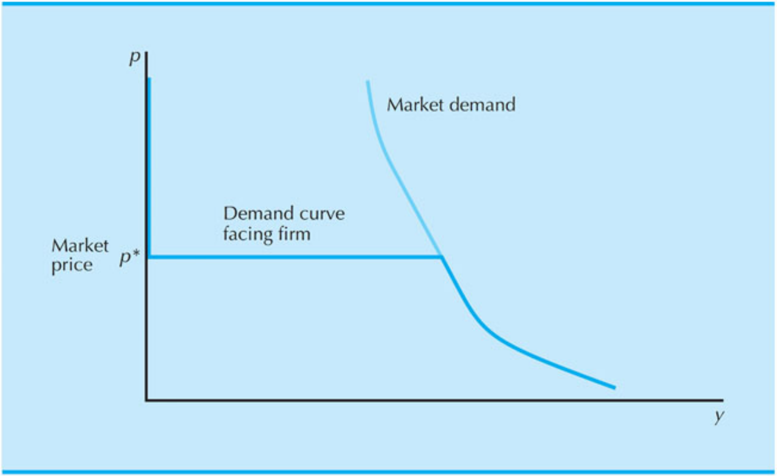
- We call the relationship between the price the firm sets and the amount it sells at that price the *demand curve faced by the firm*
- This is not the same as the market demand curve in general
  - Though that is true for monopoly
- What is the demand curve faced by the firm for perfect competition, assuming market price p^*?
  - If the firm prices above p^*, sells nothing
  - If the firm prices at or below the market price, faces the market demand curve

Demand Curve Faced by the Firm

- We call the relationship between the price the firm sets and the amount it sells at that price the *demand curve faced by the firm*
- This is not the same as the market demand curve in general
  - Though that is true for monopoly
- What is the demand curve faced by the firm for perfect competition, assuming market price p^*?
  - If the firm prices above p^*, sells nothing
  - If the firm prices at or below the market price, faces the market demand curve
- Thus under our assumption that firm is small (and hence cannot supply the whole market), they will price at the market price (if they sell at all)
- So we only have to worry about quantity decision, not pricing decision

Demand Curve Faced by Firm in Competitive Market

Figure 23.1



Supply Decision

- Assuming a cost function $c(y)$, the firm solves

$$\max_y py - c(y)$$

- The first order condition gives us

$$p = c'(y) = MC(y)$$

- That is, price (which equals marginal revenue) equals marginal cost

Supply Decision

- Assuming a cost function $c(y)$, the firm solves

$$\max_y py - c(y)$$

- The first order condition gives us

$$p = c'(y) = MC(y)$$

- That is, price (which equals marginal revenue) equals marginal cost
- The FOC gives us a relationship between market price and firm supply
- Thus $p = MC(y)$ defines the supply curve

Supply Decision

- Assuming a cost function $c(y)$, the firm solves

$$\max_y py - c(y)$$

- The first order condition gives us

$$p = c'(y) = MC(y)$$

- That is, price (which equals marginal revenue) equals marginal cost
- The FOC gives us a relationship between market price and firm supply
- Thus $p = MC(y)$ defines the supply curve
  - with just two wrinkles ...

Second-Order Condition

- Note that if the MC curve is downward sloping initially, we may have two points where $p = MC(y)$
- The lower of these two will not be maximizing, since at this point the firm could increase output to increase profits
- We can check that we are at a maximum, not a minimum, by the second-order condition, which is in this case

$$\frac{d^2\pi}{dy^2} = -c''(y) < 0$$

Shutdown Condition

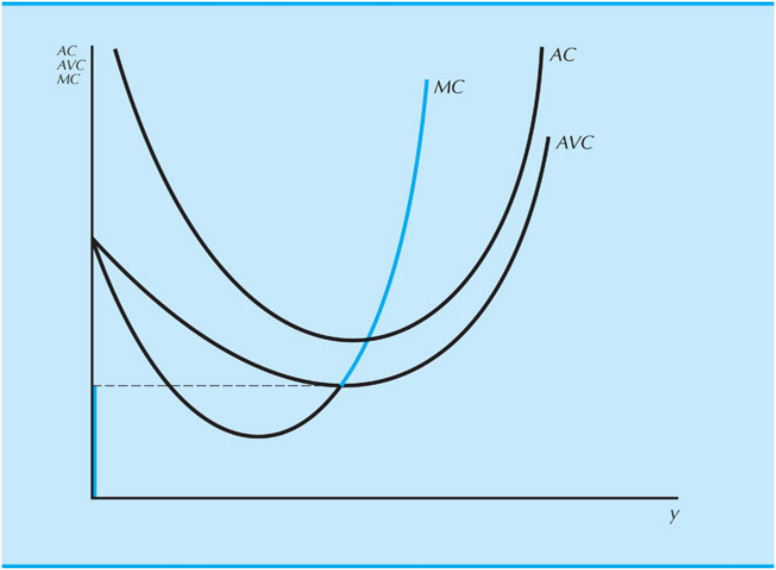
- So far we have assumed that it is optimal for firm to produce some positive amount
- Firm always has the option to shut down and produce nothing
- In this case they would pay only fixed cost F, so profit would be $-F$
- Should shut down if $py^* - c_v(y^*) - F < -F$, ie $py^* - c_v(y^*) < 0$
- Rearrange:
$$p < AVC(y^*)$$
- This is called the *shutdown condition*: firm should shut down if price is less than average variable cost at optimal quantity

Supply Curve

- Shutdown condition kicks in when MC curve goes below AVC curve
- Thus the *supply curve* is given by the part of the MC curve that upward sloping and above the AVC curve
  - If $p < \min AVC$ then the supply is zero
- As usual we can define the *inverse supply curve* as the price as a function of quantity supplied

Supply Curve Graphically

Figure 23.3



Profit

- Suppose firm is facing price p^* and producing optimal quantity y^*
- Profit is given by revenue minus costs:

$$\pi^* = p^* y^* - c(y^*) = p^* y^* - AC(y^*) y^* = [p^* - AC(y^*)] y^*$$

- This is area of rectangle of height $p^* - AC(y^*)$ and length y^*
- Note we can also write $\pi^* = p^* y^* - c_v(y^*) - F$

Profit Graphically

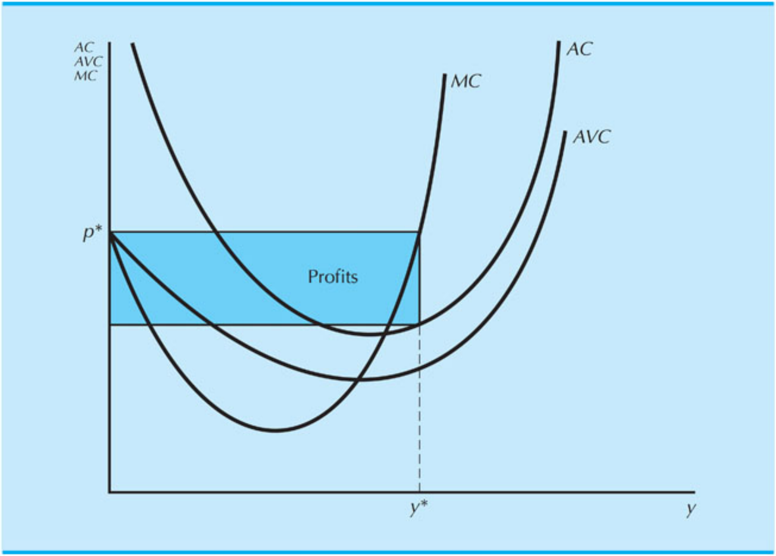


Figure 23.4

Producer Surplus

- We defined *producer surplus* as the area under the market price and above/left of the supply curve
- There is an equivalent definition: area of width y^* and height equal to the difference between $MC(y^*)$ and $AVC(Y^*)$
- That is,

$$\begin{aligned} PS =& [MC(y^*) - AVC(y^*)]y^* \\ =& \left[p^* - \frac{c_v(y^*)}{y^*}\right] y^* \\ =& p^* y^* - c_v(y^*) \end{aligned}$$

- Notice that $PS = \pi + F$, which implies $\Delta PS = \Delta\pi$

Different Ways to Calculate Producer Surplus

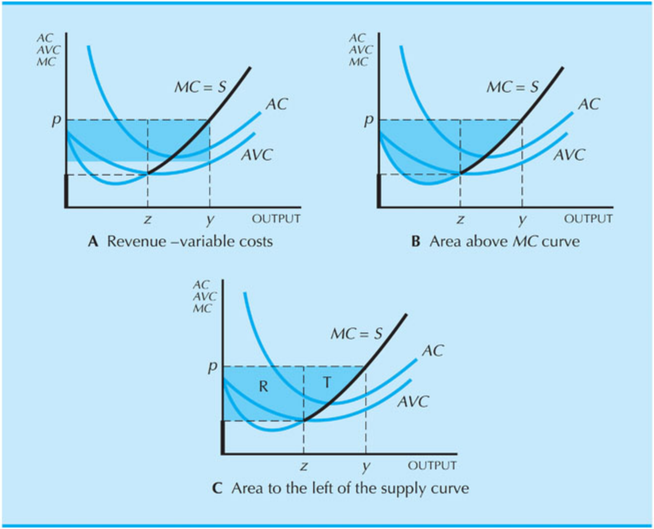


Figure 23.5

Example

- Recall our example cost function $c(y) = y^2 + 1$
- What is the supply curve?

Example

- Recall our example cost function $c(y) = y^2 + 1$
- What is the supply curve? $S(p) = \frac{p}{2}$

Example

- Recall our example cost function $c(y) = y^2 + 1$
- What is the supply curve? $S(p) = \frac{p}{2}$
- What is the inverse supply curve?

Example

- Recall our example cost function $c(y) = y^2 + 1$
- What is the supply curve? $S(p) = \frac{p}{2}$
- What is the inverse supply curve? $P(y) = 2y$

Example

- Recall our example cost function $c(y) = y^2 + 1$
- What is the supply curve? $S(p) = \frac{p}{2}$
- What is the inverse supply curve? $P(y) = 2y$
- What is profit as a function of p?

Example

- Recall our example cost function $c(y) = y^2 + 1$
- What is the supply curve? $S(p) = \frac{p}{2}$
- What is the inverse supply curve? $P(y) = 2y$
- What is profit as a function of p?

$$\pi = py - c(y) = \frac{p^2}{2} - \left(\frac{p}{2}\right)^2 - 1 = \frac{p^2}{4} - 1$$

Example

- Recall our example cost function $c(y) = y^2 + 1$
- What is the supply curve? $S(p) = \frac{p}{2}$
- What is the inverse supply curve? $P(y) = 2y$
- What is profit as a function of p?

$$\pi = py - c(y) = \frac{p^2}{2} - \left(\frac{p}{2}\right)^2 - 1 = \frac{p^2}{4} - 1$$

- What is producer surplus as a function of p?

Example

- Recall our example cost function $c(y) = y^2 + 1$
- What is the supply curve? $S(p) = \frac{p}{2}$
- What is the inverse supply curve? $P(y) = 2y$
- What is profit as a function of p?

$$\pi = py - c(y) = \frac{p^2}{2} - \left(\frac{p}{2}\right)^2 - 1 = \frac{p^2}{4} - 1$$

- What is producer surplus as a function of p?

$$PS = \frac{1}{2}p \cdot \frac{p}{2} = \frac{p^2}{4}$$

as expected from the relation $PS = \pi + F$

Appendix

Marginal Cost and Variable Cost

- There is one more property of marginal cost that comes in handy
- The area under the MC curve up until output y is equal to $VC(y)$

Marginal Cost and Variable Cost

- There is one more property of marginal cost that comes in handy
- The area under the MC curve up until output y is equal to $VC(y)$
- Intuition: adding up the marginal cost to product output y captures the variable cost up until that point, but not the fixed cost, ie the starting point
- The formal proof relies on the fundamental theorem of calculus:

$$\int_0^y MC(y')dy' = \int_0^y \frac{dc_v(y')}{dy'}dy' = c_v(y) - c_v(0) = c_v(y)$$