Econ 301: Microeconomic Analysis

Prof. Jeffrey Naecker

Wesleyan University

Labor Supply

Setting Up the Problem

- Suppose you have the utility function $U(C, H) = C^2 H$
  - C is amount of composite consumption good per day, price p
  - H is leisure time, in hours per day
- Can earn wage w per hour working out of possible hours $\bar{H}$ in the day
- Call L the hours you choose to work; remainder is leisure hours H
- Only source of income is from wages

Budget Constraint

- What is the budget constraint that you face?
- What is opportunity cost of one hour of leisure in terms of the consumption good?

Leisure Demand

- How much leisure will you demand per day?

Leisure Demand, con't

- Solve to find
  - Leisure demanded $H =$
  - Consumption demanded $C =$
- How does leisure demand depend on price of leisure (ie wage)?

Income and Substitution Effects

Motivation

- We just derived that for Cobb-Douglas utility function, hours you work does not depend on your wage
- Suppose you have a job that pays \$10/hr and you decide to work 8 hours/day
- Then suppose your wage goes up to \$12/hr
  - Do you continue to work 8 hours/day?
- What if your wage goes up to \$100/hr? \$100,000/hr?

Decomposing Demand

- Suppose the price of good 1 goes down
- Two effects on consumption decision
  1. Substitution effect
     - Have give up less of good 2 to get same amount of good 1
     - Causes more consumption of good 1 and less consumption of good 2 (holding purchasing power fixed)
  2. Income effect
     - Lower price of good 1 means more purchasing power overall
     - Causes more consumption of both goods (assuming goods are normal)

Graphical Decomposition

- Start with prices p_1, p_2, budget m, and demand $X = (x_1, x_2)$
- Want to find out what new demand will be at when price of good 1 changes to $p_1' < p_1$
- First, pivot budget curve around X until has slope $-\frac{p_1'}{p_2}$
  - Note this will require lowering income to m'
  - Optimal bundle will be at some point Y
- Second, shift budget curve such until it is back to original income level
  - That is, vertical intercept back to $\frac{m}{p_2}$
  - Optimal bundle will be at some point Z
- The move from X to Y is the *substitution effect*
- The move from Y to Z is the *income effect*

Graphical Decomposition

Substitution Effect

- This is the "pivot" part
- Change relative prices, and also change income so that consumer is able to purchase original bundle
- How much should adjusted m' income be?
  - x_1, x_2 is affordable at prices p_1', p_2 with income m': $p_1'x_1 + p_2x_2 = m'$
  - x_1, x_2 is affordable at prices p_1, p_2 with income m: $p_1x_1 + p_2x_2 = m$
  - Together these equations imply $m' - p_1'x_1 = m - p_1x_1$
  - If we define $\Delta m = x_1(p_1' - p_1) = x_1\Delta p_1$, this implies $m' = m + \Delta m$
  - Note that Δm has same sign as Δp_1
- We can then define the substitution effect as

$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m)$$

Sign of the Substitution Effect

- Consider a decrease of p_1
- Will the substitution effect increase or decrease demand of good 1?

- In general, we have $\frac{\Delta x_1^s}{\Delta p_1} \leq 0$
  - In calculus terms, that is $\frac{\partial x_1^s}{\partial p_1} \leq 0$

Income Effect

- This is the "shift" part
- To move from intermediary consumption Y to final consumption Z, imagine raising the income from m' back to m (keeping price same)
- We can define the income effect as

$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m')$$

- What sign will income effect take?

Total Change in Demand

- Note that the total change in demand is

$$\begin{aligned}\Delta x_1 &= x_1(p_1', m) - x_1(p_1, m)\\ &= x_1(p_1', m') - x_1(p_1, m) + x_1(p_1', m) - x_1(p_1', m')\\ &= \Delta x_1^s + \Delta x_1^n\end{aligned}$$

- This is one form of the *Slutsky identity* or *Slutsky equation*