Econ 301: Microeconomic Analysis

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Revealed Preference

Observing Choices

- So far in class: start with preferences/utility function, and derive choices
- Can we go the other way?
  - That is, can we derive preferences from observing choices?
- Two important assumptions throughout this section:
  1. Preferences are stable
  2. Strict convexity of preference (so unique maximum exists)

Revealed Preference

- Suppose we observe $X = (x_1, x_2)$ chosen when $Y = (y_1, y_2)$ was available at prices p_1, p_2

Definition

If X is chosen at prices p_1, p_2 and $p_1 y_1 + p_2 y_2 \leq p_1 x_1 + p_2 x_2$, then we say X is *directly revealed preferred* to Y.

- Often write this as X DRP Y
- A better term: X is *chosen over* Y

Theorem

If X DRP Y, then $X \succ Y$.

- That is, if we see X chosen over Y, we should be able to infer that $X \succ Y$

Chains of Revealed Preference

- What if we *also* observe $Y = (y_1, y_2)$ being chosen over $Z = (z_1, z_2)$ at different prices q_1, q_2?

Definition

If X is directly revealed preferred to Y and Y is directly revealed preferred to Z, then we say X is *indirectly revealed preferred* to Z, or X IRP Z.

- Note $X \succ Y$ and $Y \succ Z$ from theorem, which implies $X \succ Z$ by transitivity

Theorem

If X IRP Z, then $X \succ Z$.

Definition

If X DRP Z or X IRP Z, we say X is *revealed preferred* Z, or X RP Z.

Revealed Preference Graphically

What This Buys Us

- What do we get from setting up all these definitions?
- Two really nice tricks to go from choices to preferences:
  1. We can put bounds on indifference curves by observing choices
  2. We can check whether observed choices are consistent with maximizing behavior

Recovering Preferences

- If we observe that X DRP S under budget 1 and S DRP T under budget 2, which bundles can we compare to X?
- If we further observe Y DRP X and Z DRP X, which bundles can we compare to X?
- In observing just 4 choices, we have put strong restrictions on where indifference curve through X can lie

Recovering Preferences Graphically

Testing Choices: The Weak Axiom

- Suppose we observe X DRP Y from one budget set and Y DRP X from another budget set
  - Implies $X \succ Y$ and $Y \succ X$, a contradiction
  - Clearly such data would imply that consumer is not maximizing

Definition

The *Weak Axiom of Revealed Preference (WARP)* states that if X is directly revealed preferred to Y, then we cannot have Y directly revealed preferred to X.

Theorem

If WARP is violated, then we can conclude that observed behavior is not consistent with maximizing model of consumer choice

Testing WARP

- Given data on prices and bundles chosen at those prices, how to we check for WARP violations?
- We use the following algorithm:
  - Calculate expenditure for each bundle at each possible price
    - Expenditure is $p_1x_1 + p_2x_2$
  - Put cost of bundle i at price point j in row i, column j in matrix, ie cell (i, j)
  - Direct revealed preference when an off-diagonal entry is cheaper than then on-diagonal entry in the same column
  - Indicate direct revealed preference with $*$
  - If $*$ in at cell (r, c) and (c, r), violation of WARP

Testing WARP: Example

- Suppose we observe the following three choices under the given prices:

Choice Number	p_1	p_2	x_1	x_2
1	1	2	1	2
2	2	1	2	1
3	1	1	2	2

- Is there a violation of WARP?

Testing Choices: The Strong Axiom

- The weak axiom works only one way:
  - If we have a violation, then consumer is not maximizing
  - But it we don't find a violation, we can't be sure if consumer is maximizing
  - That is, satisfying the weak axiom is necessary but not sufficient for maximizing behavior
- Luckily we have another condition that is both necessary and sufficient for maximizing behavior:

Definition

The *Strong Axiom of Revealed Preference (SARP)* states that if X is revealed preferred (directly or indirectly) to Y, then Y cannot be revealed preferred to X (directly or indirectly).

Index Numbers

Quantity Indices

- Suppose we want to compare average purchasing behavior today (period t) to average purchasing behavior in some baseline year (period b)
- Needs weights w_1 and w_2 on goods 1 and 2:

$$I_q = \frac{w_1 x_1^t + w_2 x_2^t}{w_1 x_1^b + w_2 x_2^b}$$

- Weights tell us relative importance of the two good when evaluations how well off consumer is

Paasche quantity index

- Let weights be today's prices
- This is called *Paasche quantity index*

$$P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$

- If $P_q > 1$, what can we say about how well off consumer is today vs baseline?
- If $P_q < 1$, what can we say about how well off consumer is today vs baseline?

Laspeyres quantity index

- Let weights be baseline year's prices
- This is called *Laspeyres quantity index*

$$L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

- If $L_q < 1$, what can we say?
- If $L_q > 1$, what can we say?

Price Indices

- Quantity indices compare average consumption in two periods
- What if we want to compare average prices?
  - Use price indices:

$$I_p = \frac{p_1^t w_1 + p_2^t w_2}{p_1^b w_1 + p_2^b w_2}$$

Paasche Price Index

- Use today's consumption as weight:

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t}$$

- What can we say if $P_p < 1$?
- Define relative change in income:

$$M = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

- If $P_p > M$, what can we say?
- If $P_p < M$, what can we say?

Laspeyres Price Index

- Use baseline period's consumption as weight:

$$L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b}$$

- If $L_p < M$, what can we say?
- If $L_p > M$, what can we say?