

## Consumption Bundles

- ▶ Goods: things you can own or consume
- ▶ Consumption bundles: combinations of goods
  - ▶ Can in theory be very large, ie many goods
  - ▶ Simplify: two goods called 1 and 2 in amounts  $x_1$  and  $x_2$ , respectively
  - ▶ Consumption bundle notation:  $X = (x_1, x_2)$  or  $Y = (y_1, y_2)$ , eg
    - ▶ Capital letters for bundles
    - ▶ Subscript numbers indicate which good
- ▶ Example consumption bundles:
  - ▶ (1 car, 5 bananas)
  - ▶ (3 left shoes, 2 right shoes)
  - ▶ (2 hot dogs, 1.5 slices of pizza)

## Preferences

- ▶ The simplest comparison we can make is between two bundles
- ▶ So we need a *binary relation*
- ▶ We have a few options:
  - ▶  $X \succ Y$  means  $X$  is *strictly preferred* to  $Y$
  - ▶  $X \sim Y$  means  $X$  is *equivalent* to  $Y$  (usually say agent is *indifferent*)
  - ▶  $X \succeq Y$  means  $X$  is *weakly preferred* or  $X$  *at least as good as*  $Y$
- ▶ By convention, we usually use  $\succeq$  as our fundamental relation
  - ▶ Note we can derive the others from it
  - ▶ If  $X \succeq Y$  and  $Y \succeq X$ , then  $X \sim Y$
  - ▶ If  $X \succeq Y$  and *not*  $Y \succeq X$ , then  $X \succ Y$

## Properties of Rational Preferences Relations

- ▶ We can compare any two bundles:

### Definition

A preference relation is *complete* if for any bundles  $X$  and  $Y$ , we have  $X \succeq Y$ ,  $Y \succeq X$ , or both

- ▶ Note this means compare any bundle to itself (mostly for technical reasons)
- ▶ We also need the ordering to be logically consistent:

### Definition

A preference relation is *transitive* if for any distinct bundles  $X$ ,  $Y$ , and  $Z$ , if  $X \succeq Y$  and  $Y \succeq Z$ , then we have  $X \succeq Z$

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## Example

- ▶ Consider a choice set of  $\{A, B, C\}$  for apples, bananas, and carrots
- ▶ Suppose we have the following preference relation between the three bundles

$$\begin{array}{lll} A \succeq A & A \succeq B & A \succeq C \\ B \succeq B & B \succeq C & C \succeq C \end{array}$$

- ▶ We can represent this relation with a matrix like this:

$$\begin{array}{ccc} & A & B & C \\ A & \cdot & \cdot & \cdot \\ B & & \cdot & \cdot \\ C & & & \cdot \end{array}$$

where the  $\cdot$  in the first row, second column means that  $A \succeq B$ , etc

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## Why These Axioms?

- ▶ Without these, there might be no “best option” from a list of consumption bundles
  - ▶ Incomplete: Might not be able to pick preferred bundle out of set  $\{X, Y\}$
  - ▶ Intransitive: Even if we can make all pairwise comparisons, might not be able to choose from set of 3
- ▶ Note: these are *axioms* (guaranteed to hold) of the theory but *assumptions* (may or may not hold) about behavior

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## Example, con't

$$\begin{array}{ccc} & A & B & C \\ A & \cdot & \cdot & \cdot \\ B & & \cdot & \cdot \\ C & & & \cdot \end{array}$$

- ▶ Is this relation complete?
- ▶ Is this relation transitive?

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## Indifference Curves

## Indifference Curves

Suppose we plot all possible bundles on the cartesian grid

### Definition

The *weakly preferred set* for some bundle  $(\hat{x}_1, \hat{x}_2)$  is the set of all  $(x_1, x_2)$  such that  $(x_1, x_2) \succeq (\hat{x}_1, \hat{x}_2)$

### Definition

The *indifference curve* for some bundle  $(\hat{x}_1, \hat{x}_2)$  is the set of all  $(x_1, x_2)$  such that  $(x_1, x_2) \sim (\hat{x}_1, \hat{x}_2)$

- ▶ This is the edge of the weakly preferred set

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## Weakly Preferred Sets and Indifference Curves

## Crossing Indifference Curves

- ▶ Can indifference curves cross?

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## Monotonicity

### Definition

A preference relation satisfies *monotonicity* if  $y_1 \geq x_1$  and  $y_2 \geq x_2$  imply  $(y_1, y_2) \succeq (x_1, x_2)$

- ▶ Intuition: more is better
- ▶ What does monotonicity imply about indifference curves?

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## Utility

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## Convexity

### Definition

A preference relation satisfies *convexity* if for any bundles  $(x_1, x_2) \sim (y_1, y_2)$  and any  $\alpha \in [0, 1]$  we have  $(\alpha y_1 + (1 - \alpha)x_1, \alpha y_2 + (1 - \alpha)x_2) \succeq (x_1, x_2)$

- ▶ Intuition: averages are better
- ▶ What does convexity imply about weakly preferred sets?

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## Utility

### Definition

An *utility function* is a function  $u$  that assigns a real number to every bundle such that  $u(x_1, x_2) \geq u(y_1, y_2)$  if and only if  $(x_1, x_2) \succeq (y_1, y_2)$

- ▶ Derived from preferences, which are considered the fundamental object
- ▶ Preferences only tell us the order or ranking of bundles, so we usually talk about *ordinal* utility

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## Existence of Utility Functions

- ▶ Does a utility function exist for every possible preference ranking?
- ▶ Assuming complete and transitive preferences, a utility function (or really functions) will exist
- ▶ A simple construction of a utility function: label all indifference curves with increasing numbers
  - ▶ Indifference curves are *level sets* utility functions
  - ▶ That is, an indifference curve is the set of points  $(x_1, x_2)$  such that  $u(x_1, x_2) = k$  for some  $k$

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## A Family of Utility Functions

- ▶ All of these utility functions  $U_1$  through  $U_4$  represent the same preferences:

bundle	$U_1$	$U_2$	$U_3$	$U_4$
A	3	4	17.85	-1
B	2	3	11.20	-2
C	1	2	0.01	-3

- ▶ All we are doing is relabeling indifference curves
- ▶ From one preference ordering we can arrive at many utility functions
- ▶ This is why we only concern ourselves with *ordinal* information in utility functions

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## Monotone Transformations

### Definition

A function  $f$  is *monotonic* if  $f(u_2) - f(u_1) \geq 0$  for any  $u_1$  and  $u_2$  such that  $u_2 \geq u_1$ . That is, it is *weakly increasing*.

### Theorem

If  $f(u)$  is a monotonic function, and  $u(x_1, x_2)$  is a utility function representing some preference ordering, then  $f(u(x_1, x_2))$  is also a utility function representing that same preference ordering

What does this mean practically?

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## Marginal Rates of Substitution

- ▶ The marginal rate of substitution (MRS) from good 1 to good 2 is the exchange rate at which the consumer would be just indifferent about trading
- ▶ In terms of indifference curves: MRS is the slope of the indifference curve (may be changing)
- ▶ In terms of the utility function:

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -\frac{MU_1}{MU_2}$$

where  $MU_i = \frac{\partial u}{\partial x_i}$  is called the *marginal utility* of good  $i$

- ▶ We usually have *diminishing marginal rates of substitution*

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## Examples

### Perfect Substitutes (1-to-1)

- ▶ Preference relation: consumer is equally happy to swap one good for the other at rate of 1-to-1
- ▶ Example?
- ▶ Indifference curve:

- ▶ Utility function:
- ▶ MRS =

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### Perfect Complements (1-to-1)

- ▶ Preference relation: consumer prefers to consume goods together in 1-to-1 proportion
- ▶ Example?
- ▶ Indifference curve:

- ▶ Utility function:
- ▶ MRS =

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### Cobb-Douglas

- ▶ Utility function:  $u(x_1, x_2) = x_1^c x_2^d$  for  $c, d > 0$
- ▶ Indifference curve:
- ▶ MRS =
- ▶ Very useful for approximating preferences in the real world

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