# Econ 301: Microeconomic Analysis

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Exchange

# Motivation

- So far in this class we have looked at one market at a time
  - Equilibrium in just one market (ignoring all others) is called *partial equilibrium*
- But in general what happens in one market will affect outcomes in other markets
  - So we move to study *general equilibrium*, which is equilibrium of all markets in the economy at the same time

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- But in general what happens in one market will affect outcomes in other markets
  - So we move to study *general equilibrium*, which is equilibrium of all markets in the economy at the same time
- Simplifying assumptions:
  - Fully competitive markets
  - Just two markets and two consumers
  - Focus on *pure exchange* for now: trade with no production

# Edgeworth Box: Setup

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- An allocation $(x_A, x_B)$ is *feasible* if $x_A^1 + x_B^1 = \omega_A^1 + \omega_B^1$ and $x_A^2 + x_B^2 = \omega_A^2 + \omega_B^2$

# Drawing the Edgeworth Box

- Width of box: total amount of good 1 in economy: $\omega_A^1 + \omega_B^1$
- Height of box: total amount of good 2 in economy: $\omega_A^2 + \omega_B^2$
- Endowment $W = (\omega_A, \omega_B)$ is a point in the box
- Consumer A's allocation measured from lower left corner, which consumer B's endowment measured from upper right corner
- Consumer A's indifference curves open up and to the right, while consumer B's indifference curves open down and to the left

# Edgeworth Box

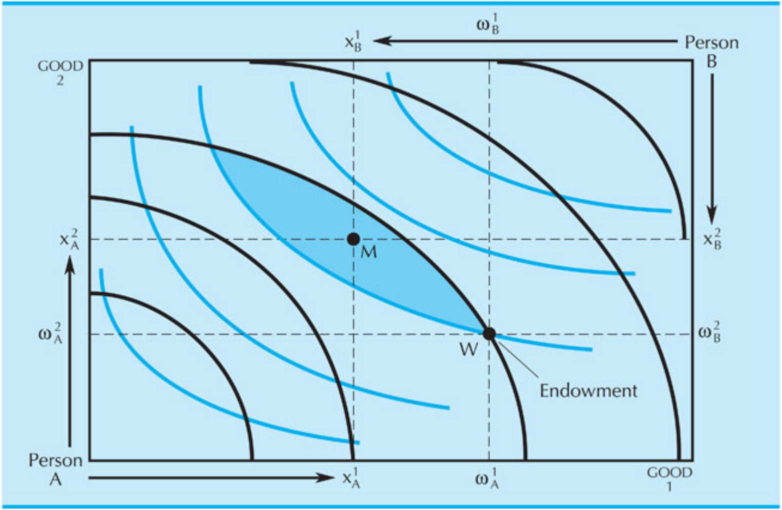


Figure 32.1

# Trade in the Edgeworth Box

- Suppose that the consumers start at a point $W = (\omega_A, \omega_B)$ in the box
- Remember, an allocation is *Pareto efficient* if no one can be made better off without making someone worse off
- Is the endowment point is Pareto efficient?

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- Remember, an allocation is *Pareto efficient* if no one can be made better off without making someone worse off
- Is the endowment point is Pareto efficient?
  - Draw the indifference curves for both consumers that go through $W$
  - In general, there will be a lens-shaped area that is above A's indifference curve and below B's indifference curve
    - In this area, both consumers are better off than at endowment
  - Any trade that occurs should move consumers to a point in this region
  - Consumers are both better off anywhere in lens, so endowment is *not* Pareto efficient

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  - Yes, in general there will be a continuum of Pareto efficient points
  - We call this the *contract curve*
  - Note that the bottom left and upper right corners must be on the contract curve
- Thus any trade starting at the endowment must end up on the contract curve and inside the lens
  - We call this part of the contract curve the *core*

# Contract Curve

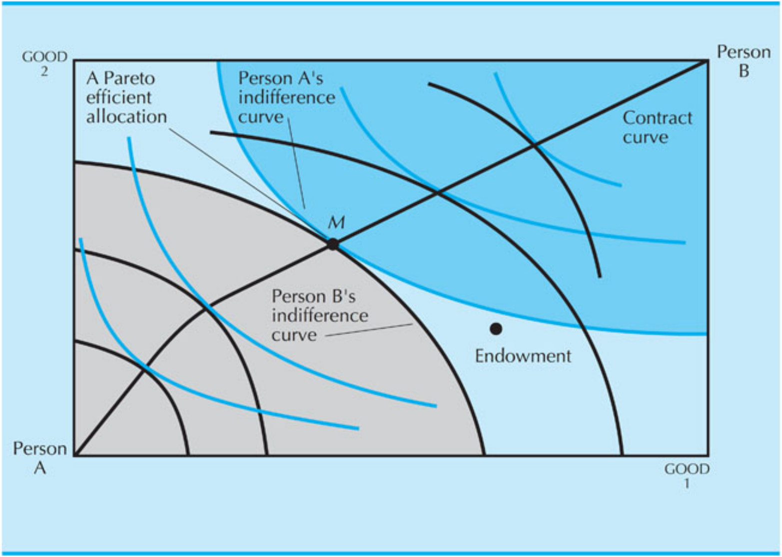


Figure 32.2

# Adding Prices

- Imagine a neutral third party (often called the *auctioneer*) who sets prices $p = (p_1, p_2)$ for goods 1 and 2
- Based on preferences and budget, we can calculate each consumer's demand (sometimes called *gross demand*):

$$x_A = x_A(p, m_A) = (x_A^1(p, m_A), x_A^2(p, m_A))$$
$$x_B = x_B(p, m_B) = (x_B^1(p, m_B), x_B^2(p, m_B))$$

- We then define *excess* or *net demand* for each consumer:

$$e_A = (e_A^1, e_A^2) = (x_A^1 - \omega_A^1, x_A^2 - \omega_A^2)$$
$$e_B = (e_B^1, e_B^2) = (x_B^1 - \omega_B^1, x_B^2 - \omega_B^2)$$

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\begin{aligned}
p_1 x_A^1 + p_2 x_A^2 &= p_1 \omega_A^1 + p_2 \omega_A^2 \\
\Longrightarrow \; p_1(\omega_A^1 + \omega_B^1 - x_B^1) + p_2(\omega_A^2 + \omega_B^2 - x_B^2) &= p_1 \omega_A^1 + p_2 \omega_A^2 \\
\Longrightarrow \; p_1 x_B^1 + p_2 x_B^2 &= p_1 \omega_B^1 + p_2 \omega_B^2
\end{aligned}
$$

- Note that the endowment point is on the budget set implied by the prices: consumer could decide to just consume their endowment

# Demand in the Edgeworth Box

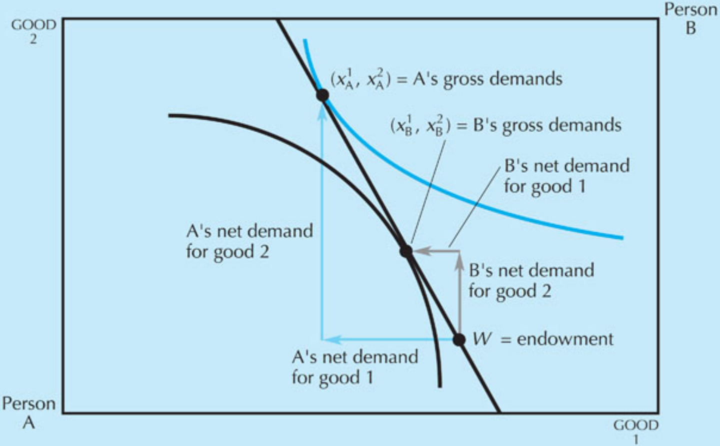


Figure 32.3

# Competitive Equilibrium

- The economy is in *competitive equilibrium* (or *Walrasian equilibrium*) at prices $p^* = (p_1^*, p_2^*)$ and endowment $W = (\omega_A, \omega_B)$ when total demand equals total supply in each market
  - For 2-good economy, this means we have

$$x_A^1(p^*) + x_B^1(p^*) = \omega_A^1 + \omega_B^1$$
$$x_A^2(p^*) + x_B^2(p^*) = \omega_A^2 + \omega_B^2$$

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- Note that since both consumers are optimizing, their indifference curves must be tangent to budget curve
  - And since they face the same prices, the indifference curves must also be tangent to each other
- Equilibrium is guaranteed to exist (as long as each consumer's demand is continuous, or each consumer is small relative to the market)

# Aggregate Excess Demand

- We can define the *aggregate excess demand* for each good

$$z_1(p) = \underbrace{x_A^1(p) - \omega_A^1}_{e_A^1(p)} + \underbrace{x_B^1(p) - \omega_B^1}_{e_B^1(p)}$$

$$z_2(p) = \underbrace{x_A^2(p) - \omega_A^2}_{e_A^2(p)} + \underbrace{x_B^2(p) - \omega_B^2}_{e_B^2(p)}$$

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- This gives us a new definition of competitive equilibrium:

$$z_1(p^*) = 0$$
$$z_2(p^*) = 0$$

- That is, aggregate excess demand of each good must be zero
- Each consumer wants to buy exactly as much as the other is selling (or vice versa)

# Walras's Law

- Note consumer A's budget constraint can be written as

$$p_1 x_A^1 + p_2 x_A^2 = p_1 \omega_A^1 + p_2 \omega_A^2$$

- Rearranging, we get $p_1(x_A^1 - \omega_A^1) + p_2(x_A^2 - \omega_A^2) = 0$, or equivalently

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- Similarly, for B we will get $p_1 e_B^1 + p_2 e_B^2 = 0$
- Adding A and B's conditions together give

$$p_1(e_A^1 + e_B^1) + p_2(e_A^2 + e_B^2) = 0$$

  or

$$p_1 z_1 + p_2 z_2 = 0$$

- This last expression is known as *Walras's Law*

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- Note that if $z_1 = 0$ then Walras's Law gives $p_2 z_2 = 0$
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- Clearly by same logic if $z_2 = 0$ we can immediately conclude that $z_1 = 0$ (as long as $p_1 > 0$)
- This means that for equilibrium it is sufficient to check just one of the two excess demand conditions
- In general, if we have $k$ goods and $k - 1$ of them are in equilibrium, the $k$th market will be in equilibrium as well
- Note that we are therefore free to set price of one good equal to 1 (the *numeraire good*)

# Equilibrium in the Edgeworth Box

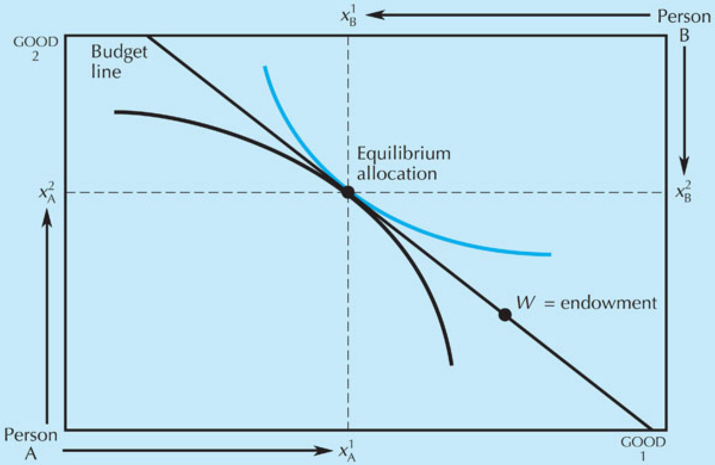


Figure
32.4

# Equilibrium Example

- Suppose both consumer have Cobb-Douglas preferences:

$$u_A(x_A^1, x_A^2) = (x_A^1)^a (x_A^2)^{1-a}$$
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- What is (gross) demand for the two consumers?

$$x_A^1(p, m_A) = a\frac{m_A}{p_1} \qquad x_B^1(p, m_B) = b\frac{m_B}{p_1}$$
$$x_A^2(p, m_A) = (1-a)\frac{m_A}{p_2} \qquad x_B^2(p, m_B) = (1-b)\frac{m_B}{p_2}$$

where $m_i = p_1\omega_i^1 + p_2\omega_i^2$ for $i = A, B$

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- Setting $p_2^* = 1$ (numeraire) and solving for $p_1$ we get

$$p_1^* = \frac{a\omega_A^2 + b\omega_B^2}{(1-a)\omega_A^1 + (1-b)\omega_B^1}$$

# Contract Curve

- What is formula for contract curve in this example?
  - Note that contract curve is where indifference curves are tangent
  - Slope of indifference curve is MRS
  - $MRS_A = \frac{a}{1-a}\frac{x_A^2}{x_A^1}$ and $MRS_B = \frac{b}{1-b}\frac{x_B^2}{x_B^1} = \frac{b}{1-b}\frac{\omega_A^2+\omega_B^2-x_A^2}{\omega_A^1+\omega_B^1-x_A^1}$
  - Setting these equal, will get equation for contral curve

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- Suppose equilibrium $(x_A, x_B, p_1, p_2)$ was <u>not</u> Pareto efficient
- Then there must exist allocation $(y_A, y_B)$ that is both feasible and desirable for both consumers:

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$$\begin{aligned} y_A^1 + y_B^1 &= \omega_A^1 + \omega_B^1 \\ y_A^2 + y_B^2 &= \omega_A^2 + \omega_B^2 \end{aligned}$$

- For $(x_A, x_B)$ to be optimal it must be that $(y_A, y_B)$ was not affordable:

$$\begin{aligned} p_1 y_A^1 + p_2 y_A^2 &> p_1 \omega_A^1 + p_2 \omega_A^2 \\ p_1 y_B^1 + p_2 y_B^2 &> p_1 \omega_B^1 + p_2 \omega_B^2 \end{aligned}$$

# First Welfare Theorem (continued)

- Adding these last two equations together we get

$$p_1(y_A^1 + y_B^1) + p_2(y_A^2 + y_B^2) > p_1(\omega_A^1 + \omega_B^1) + p_2(\omega_A^2 + \omega_B^2)$$

- Plugging in the feasibility condition we get

$$p_1(\omega_A^1 + \omega_B^1) + p_2(\omega_A^2 + \omega_B^2) > p_1(\omega_A^1 + \omega_B^1) + p_2(\omega_A^2 + \omega_B^2)$$

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- Clearly a contradiction
- Thus it must be that any competitive equilibrium is Pareto efficient
  - This is known as the *First Welfare Theorem*
- Huge implication: Market process will automatically find an efficient outcome (though not necessarily a fair one)

# Second Welfare Theorem

- OK, so the First Welfare Theorem says that a competitive equilibrium is Pareto efficient
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- OK, so the First Welfare Theorem says that a competitive equilibrium is Pareto efficient
- Is the converse true? That is, are all Pareto efficient allocations possible equilibria?
- Yes, any Pareto efficient allocation can be a competitive equilibrium for some prices $p$ and endowments $W$
  - This is the *Second Welfare Theorem*
  - Guaranteed as long as preferences are convex
  - Intuition: for Pareto efficiency, indifference curves are tangent, so we can find prices and endowment to run a budget curve right through the tangency point
- Huge implication: To get a desired efficient market outcome, just have to choose starting endowment and let market forces do their work