Econ 301: Microeconomic Analysis

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Consumer Surplus

Motivating Example

- Quasilinear utility: $u(x, y) = v(x) + y$
- Prices $(p, 1)$ and income m
- x is a *discrete good*: can only buy integer amounts
- y is continuous good
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    - $m - pn$ units

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- Will demand n units of good 1 iff

$$r_{n+1} \leq p \leq r_n$$

Reservation Prices

- Define *reservation prices*

$$
\begin{aligned}
r_0 =& v(0) \\
r_1 =& v(1) - v(0) \\
r_2 =& v(2) - v(1) \\
& \dots \\
r_n =& v(n) - v(n-1)
\end{aligned}
$$

- Prices at which consumer is just indifferent between consuming and not consuming additional unit

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- If we subtract out the money spent on those n units, we get *net consumer surplus*:

$$v(n) - pn$$

  - We often drop the “net” and just call this *consumer surplus* (CS)
  - Intuition: extra happiness from consumption

Gross and Net Surplus in Quasilinear Case

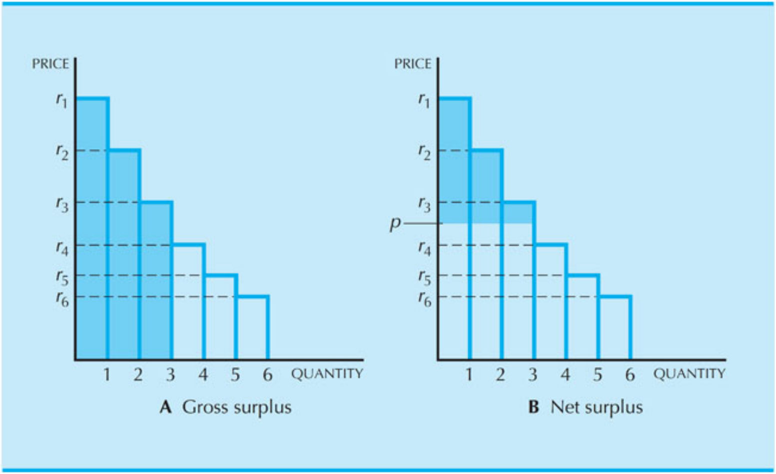

Figure 14.1

Consumer Surplus (General Case)

- In general (ie continuous goods, any utility function), we define *consumer surplus (CS)* to be the area below the demand curve and above the price

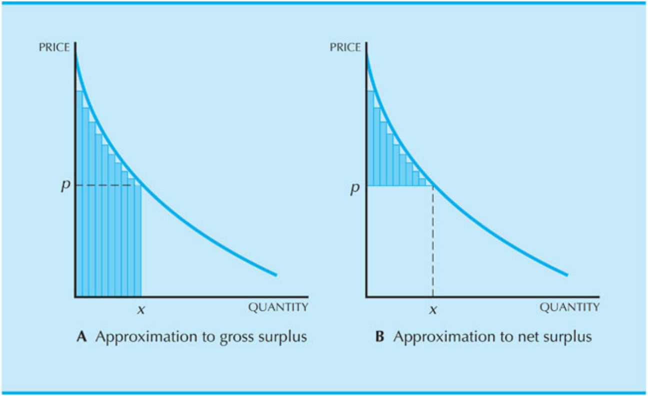

Figure 14.2

Changes in Consumer Surplus

- Typically we care about the change in consumer surplus when price increases from p' to p''
- Let the demand be given by $x(p)$ (fixing income and all other prices)
- Then we can use an integral to calculate the *change in consumer surplus*:

$$\Delta CS = \int_{p''}^{p'} x(p)dp$$

- Note that integral is over p since demand is function of price
  - Usually graph with p on the vertical axis, so integral is along *vertical* axis

Changes in Consumer Surplus, Graphically

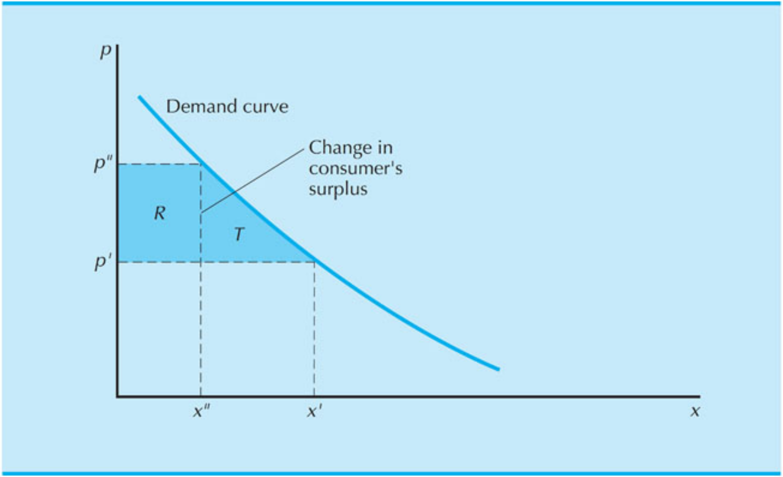

Figure 14.3

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$$\begin{aligned}
\Delta CS &= \int_1^2 x(p)dp \\
&= \int_1^2 (a - bp)dp \\
&= \left[ap - \frac{1}{2}bp^2\right]_1^2 \\
&= \left[2a - \frac{1}{2}b(2)^2\right] - \left[a - \frac{1}{2}b(1)^2\right] \\
&= a - \frac{3}{2}b
\end{aligned}$$

- Note we could also get this from formula for area of triangle

Market Demand

Adding up Individual Demand

- Suppose there are N consumers in the market, indexed $i = 1, 2, \ldots, N$
- Demand from consumer i: $x_i(p_1, p_2, m_i)$
- Define *aggregate* or market demand as

$$X(p_1, p_2, m_1, m_2, \ldots, m_N) = \sum_{i=1}^{N} x_i(p_1, p_2, m_i)$$

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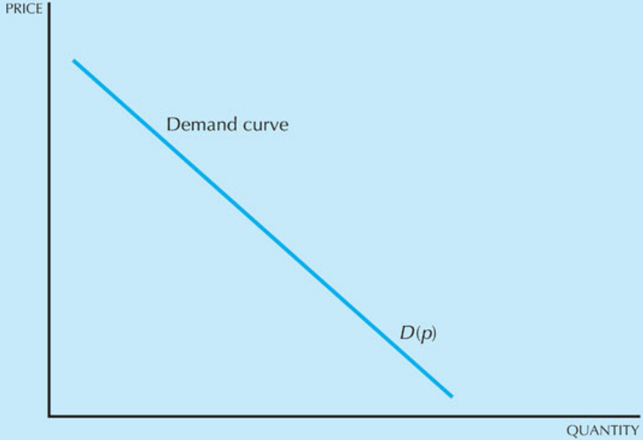
- Assume that behavior of all agents together can be summarized by behavior of one agent with income equal to sum of all incomes
  - $M = \sum_{i=1}^{N} m_i$
  - This is called *representative consumer*
- Then market demand can be written $X(p_1, p_2, M)$
  - Focusing on one good and fixing all other prices and M, we also write $D(p)$ for the demand

Market Demand Curve



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Adding Up Demand

- We can also think of price as a function of quantity, called *inverse demand*, which we write as $P(X)$ or $P(Q)$
- Demand and inverse demand are two ways of writing the same function
- However, we add up demans *horizontally* on the standard graph
- Two kinds of changes in purchase behavior:
  - Intensive margin: deciding how much to purchase
  - Extensive margin: deciding whether to purchase anything at all

Adding Up Demand: Example

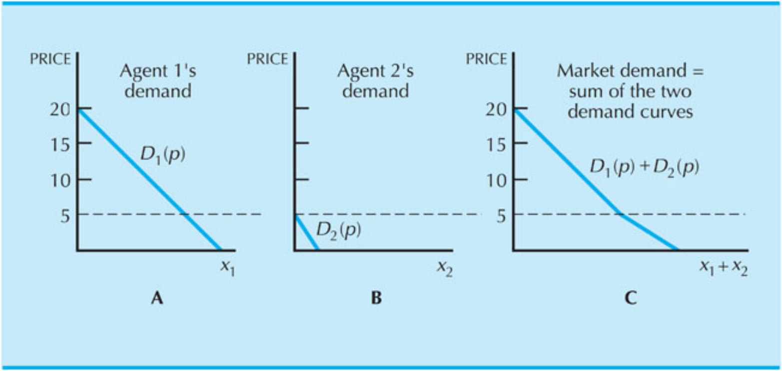

Figure 15.2

Elasticity

- We want to know how responsive demand is to price
- We can look at the slope of the demand cuve, $\frac{\Delta q}{\Delta p}$
- Problem: changing units makes comparing this number across markets difficult
- Solution: look at percent change in quantity per percent change in price:

$$\frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{p}{q}\frac{\Delta q}{\Delta p}$$

- We call this number the *elasticity*, denoted by ϵ:

$$\epsilon(p) = \frac{p}{q}\frac{dq}{dp}$$

- Note that we usually write elasticity as a function of price

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  3. If $|\epsilon| = 1$, we say demand is *unit elastic*

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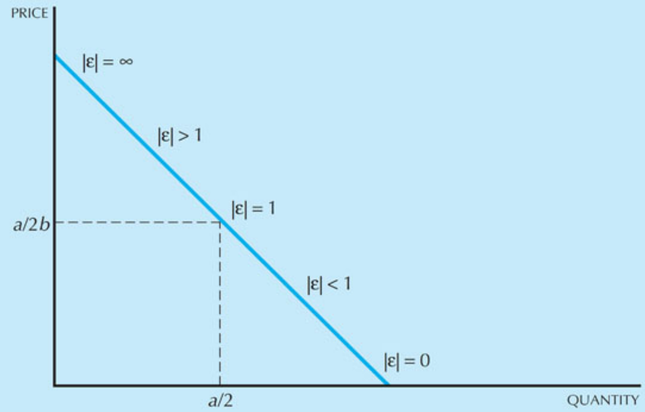
- Note that elasticity can change a lot with price
  - In example above, $|\epsilon|$ ranges from 0 to ∞

Elasticity of Linear Demand



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- Hence this is called *constant elasticity* demand
  - We sometimes write this as $q = Ap^{\epsilon}$ to remind ourselves that the exponent is just the elasticity

Appendix

Compensating and Equivalent Variation

- Consider prices $(p_1^*, 1)$ and budget m^*, which leads to demand (x_1^*, x_2^*)
- Suppose we increase price of good 1 to $\hat{p}_1$, so demand changes to $(\hat{x}_1, \hat{x}_2)$
- *Compensating variation* (CV): how far we have to shift new budget line to make consumer as well off as they were under original prices

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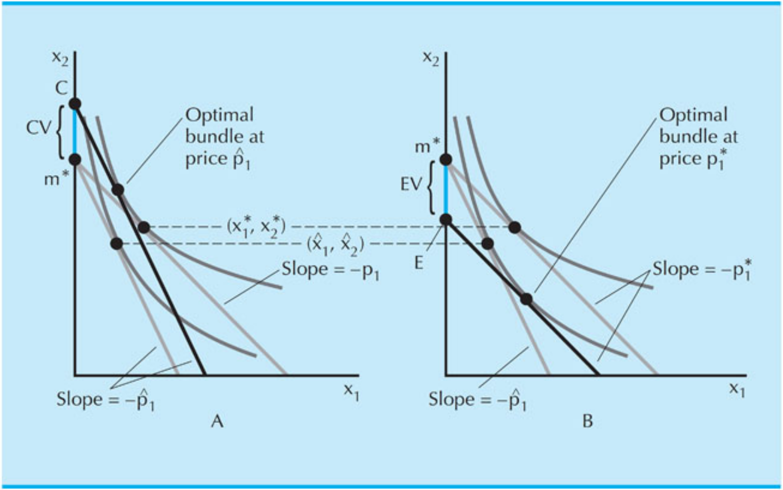
- *Compensating variation* (CV): how far we have to shift new budget line to make consumer as well off as they were under original prices
- *Equivalent variation* (EV): how far we have to shift original budget line to make consumer as well off as they are under new prices

CV and EV



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Example: Quasilinear Preferences

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  - Must have $v(\hat{x}_1) + m + CV - \hat{p}_1 \hat{x}_1 = v(x_1^*) + m - p_1^* x_1^*$
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- What is equivalent variation?
  - Must have $v(\hat{x}_1) + m - \hat{p}_1 \hat{x}_1 = v(x_1^*) + m - EV - p_1^* x_1^*$
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- What is ΔCS? $v(x_1^*) - p_1^* x_1^* - [v(\hat{x}_1) - \hat{p}_1 \hat{x}_1]$

Relationship Between CV, EV, and ΔCS

- We saw that for quasilisnear preferences $CV = \Delta CS = EV$
- In general, we have the following relationship between the three quantities:

$$CV \leq \Delta CS \leq EV$$

- This is why ΔCS is a good estimate of the welfare effects of changing prices