

Econ 301: Microeconomic Analysis

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Labor Supply

Setting Up the Problem

- ▶ Suppose you have the utility function $U(C, H) = C^2 H$
 - ▶ C is amount of composite consumption good per day, price p
 - ▶ H is leisure time, in hours per day
- ▶ Can earn wage w per hour working out of possible hours \bar{H} in the day
- ▶ Call L the hours you choose to work; remainder is leisure hours H
- ▶ Only source of income is from wages

Budget Constraint

- ▶ What is the budget constraint that you face?
 - ▶ Earn income $wL = w(\bar{H} - H)$ for working
 - ▶ Spend pC on consumption good
 - ▶ Therefore must have $pC = w(\bar{H} - H)$
 - ▶ Note we can rearrange to $pC + wH = w\bar{H}$
- ▶ What is opportunity cost of one hour of leisure in terms of the consumption good?
 - ▶ One extra hour of leisure means one less hour of work
 - ▶ That means w less income to spend
 - ▶ Could have bought $\frac{w}{p}$ units of consumption good with that income
 - ▶ Thus opportunity cost of hour of leisure is $\frac{w}{p}$

Leisure Demand

- ▶ How much leisure will you demand per day?
- ▶ Setting up constrained optimization problem:

$$\mathcal{L} = C^2 H + \lambda(w\bar{H} - pC - wH)$$

- ▶ First order conditions:

$$2CH - \lambda p = 0 \quad C^2 - \lambda w = 0 \quad w\bar{H} - wH - pC = 0$$

- ▶ Take the ratio of these to find

$$\frac{2H}{C} = \frac{p}{w}$$

- ▶ Plug into the budget constraint:

$$p \left(2H \frac{w}{p} \right) + wH = w\bar{H}$$

Leisure Demand, con't

- ▶ Solve to find
 - ▶ Leisure demanded $H = \frac{\bar{H}}{3}$
 - ▶ Consumption demanded $C = \frac{2}{3} \frac{w}{p} \bar{H}$
- ▶ How does leisure demand depend on price of leisure (ie wage)?
 - ▶ Leisure demanded (or equivalently labor supplied) does not change with wages or price of consumption
 - ▶ Eg if $\bar{H} = 24$, you will work 16 hours a day regardless of the wage
 - ▶ This is the case for Cobb-Douglas utility function, though not in general

Income and Substitution Effects

Motivation

- ▶ We just derived that for Cobb-Douglas utility function, hours you work does not depend on your wage
- ▶ Suppose you have a job that pays \$10/hr and you decide to work 8 hours/day
- ▶ Then suppose your wage goes up to \$12/hr
 - ▶ Do you continue to work 8 hours/day?
- ▶ What if your wage goes up to \$100/hr? \$100,000/hr?

Decomposing Demand

- ▶ Suppose the price of good 1 goes down
- ▶ Two effects on consumption decision
 1. Substitution effect
 - ▶ Have give up less of good 2 to get same amount of good 1
 - ▶ Causes more consumption of good 1 and less consumption of good 2 (holding purchasing power fixed)
 2. Income effect
 - ▶ Lower price of good 1 means more purchasing power overall
 - ▶ Causes more consumption of both goods (assuming goods are normal)

Graphical Decomposition

- ▶ Start with prices p_1, p_2 , budget m , and demand $X = (x_1, x_2)$
- ▶ Want to find out what new demand will be at when price of good 1 changes to $p'_1 < p_1$
- ▶ First, pivot budget curve around X until has slope $-\frac{p'_1}{p_2}$
 - ▶ Note this will require lowering income to m'
 - ▶ Optimal bundle will be at some point Y
- ▶ Second, shift budget curve such until it is back to original income level
 - ▶ That is, vertical intercept back to $\frac{m}{p_2}$
 - ▶ Optimal bundle will be at some point Z
- ▶ The move from X to Y is the *substitution effect*
- ▶ The move from Y to Z is the *income effect*

Graphical Decomposition

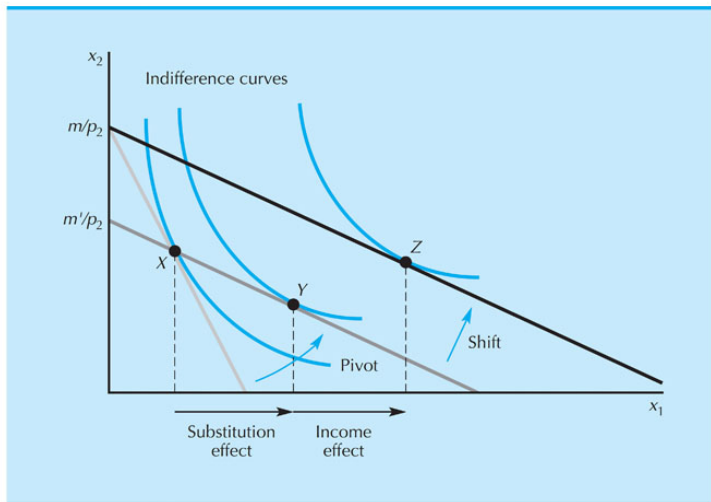


Figure
8.2

Substitution Effect

- ▶ This is the “pivot” part
- ▶ Change relative prices, and also change income so that consumer is able to purchase original bundle
- ▶ How much should adjusted m' income be?
 - ▶ x_1, x_2 is affordable at prices p'_1, p_2 with income m' : $p'_1 x_1 + p_2 x_2 = m'$
 - ▶ x_1, x_2 is affordable at prices p_1, p_2 with income m : $p_1 x_1 + p_2 x_2 = m$
 - ▶ Together these equations imply $m' - p'_1 x_1 = m - p_1 x_1$
 - ▶ If we define $\Delta m = x_1(p'_1 - p_1) = x_1 \Delta p_1$, this implies $m' = m + \Delta m$
 - ▶ Note that Δm has same sign as Δp_1
- ▶ We can then define the substitution effect as

$$\Delta x_1^S = x_1(p'_1, m') - x_1(p_1, m)$$

Sign of the Substitution Effect

- ▶ Consider a decrease of p_1
- ▶ Will the substitution effect increase or decrease demand of good 1?
 - ▶ Pivoted budget constraint will contain options to the left of original bundle
 - ▶ These were available before, yet not chosen, so original bundle is preferred to them
 - ▶ Thus intermediary bundle Y must lie on pivoted budget constraint, to the right of original bundle
 - ▶ Thus $p'_1 < p_1$ leads to $\Delta x_1^s \geq 0$
- ▶ In general, we have $\frac{\Delta x_1^s}{\Delta p_1} \leq 0$
 - ▶ In calculus terms, that is $\frac{\partial x_1^s}{\partial p_1} \leq 0$

Income Effect

- ▶ This is the “shift” part
- ▶ To move from intermediary consumption Y to final consumption Z , imagine raising the income from m' back to m (keeping price same)
- ▶ We can define the income effect as

$$\Delta x_1^n = x_1(p'_1, m) - x_1(p'_1, m')$$

- ▶ What sign will income effect take?
 - ▶ Normal good: an increase in income causes an increase in demand
 - ▶ ie, $\frac{\Delta x_1^n}{\Delta m} > 0$
 - ▶ Inferior good: an increase in income causes a decrease in demand
 - ▶ ie, $\frac{\Delta x_1^n}{\Delta m} < 0$
 - ▶ Thus the sign of $\frac{\partial x_1^n}{\partial m} \approx \frac{\Delta x_1^n}{\Delta m}$ depends on whether good 1 is normal good

Total Change in Demand

- ▶ Note that the total change in demand is

$$\begin{aligned}\Delta x_1 &= x_1(p'_1, m) - x_1(p_1, m) \\ &= x_1(p'_1, m') - x_1(p_1, m) + x_1(p'_1, m) - x_1(p'_1, m') \\ &= \Delta x_1^s + \Delta x_1^n\end{aligned}$$

- ▶ This is one form of the *Slutsky identity* or *Slutsky equation*