

# Econ 301: Microeconomic Analysis

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# The Slutsky Equation

# Total Change in Demand

- ▶ Last time we defined
  - ▶ Income effect:  $\Delta x_1^n = x_1(p'_1, m) - x_1(p'_1, m')$
  - ▶ Substitution effect:  $\Delta x_1^s = x_1(p'_1, m') - x_1(p_1, m)$
- ▶ Note that the total change in demand is

$$\begin{aligned}\Delta x_1 &= x_1(p'_1, m) - x_1(p_1, m) \\ &= x_1(p'_1, m') - x_1(p_1, m) + x_1(p'_1, m) - x_1(p'_1, m') \\ &= \Delta x_1^s + \Delta x_1^n\end{aligned}$$

- ▶ This is one form of the *Slutsky identity* or *Slutsky equation*

# Signing the Total Change in Demand

- ▶ What sign does the overall change in demand take if  $\Delta p_1 < 0$ ?
- ▶ We showed that  $\Delta x_1^s > 0$  if  $\Delta p_1 < 0$
- ▶ What if good 1 is normal?
  - ▶ Then  $\Delta x_1^n > 0$ , so  $\Delta x_1 > 0$
- ▶ What if good 1 is inferior?
  - ▶ Then  $\Delta x_1^n < 0$ , so sign of  $\Delta x_1$  is ambiguous
  - ▶ Note  $\Delta x_1 < 0$  (a Giffen good) if  $|\Delta x_1^n| > |\Delta x_1^s|$
- ▶ Are all Giffen goods inferior? Yes
- ▶ Are all inferior goods Giffen? No

# Giffen and Inferior Goods Graphically

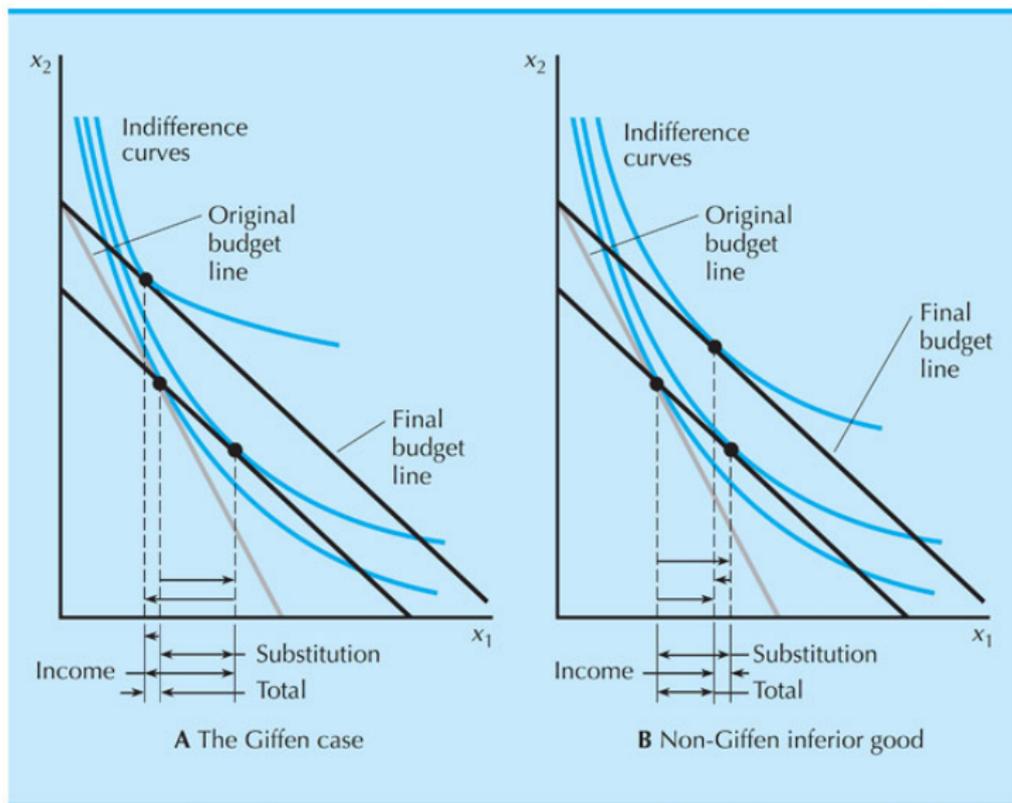


Figure 8.3

# Example

- ▶ Let demand function be given by  $x_1 = 10 + \frac{m}{10p_1}$
- ▶ Suppose we start out at  $p_1 = 3$  and  $m = 120$
- ▶ Suppose price decrease to  $p'_1 = 2$
- ▶ Substitution effect?
  - ▶ Starting demand  $x_1 = 10 + \frac{120}{30} = 14$
  - ▶  $\Delta m = x_1 \Delta p_1 = 14(-1) = -14$
  - ▶  $m' = 120 - 14 = 106$
  - ▶ Intermediate demand  $x_1(p'_1, m') = 10 + \frac{106}{20} = 15.3$
  - ▶  $\Delta x_1^s = x_1(p'_1, m') - x_1(p_1, m) = 15.3 - 14 = 1.3$
- ▶ Income effect?
  - ▶ Final demand  $x_1(p'_1, m) = 10 + \frac{120}{20} = 16$
  - ▶  $\Delta x_1^i = x_1(p'_1, m) - x_1(p'_1, m') = 16 - 15.3 = 0.7$

# Perfect Complements

- ▶ Consider perfect complements preferences
- ▶ What are the income and substitution effects of a decrease in  $p_1$ ?
  - ▶ Note that pivot has no effect on optimal consumption
  - ▶ Thus substitution effect is zero
  - ▶ Thus demand change is entirely from income effect

# Perfect Complements Graphically

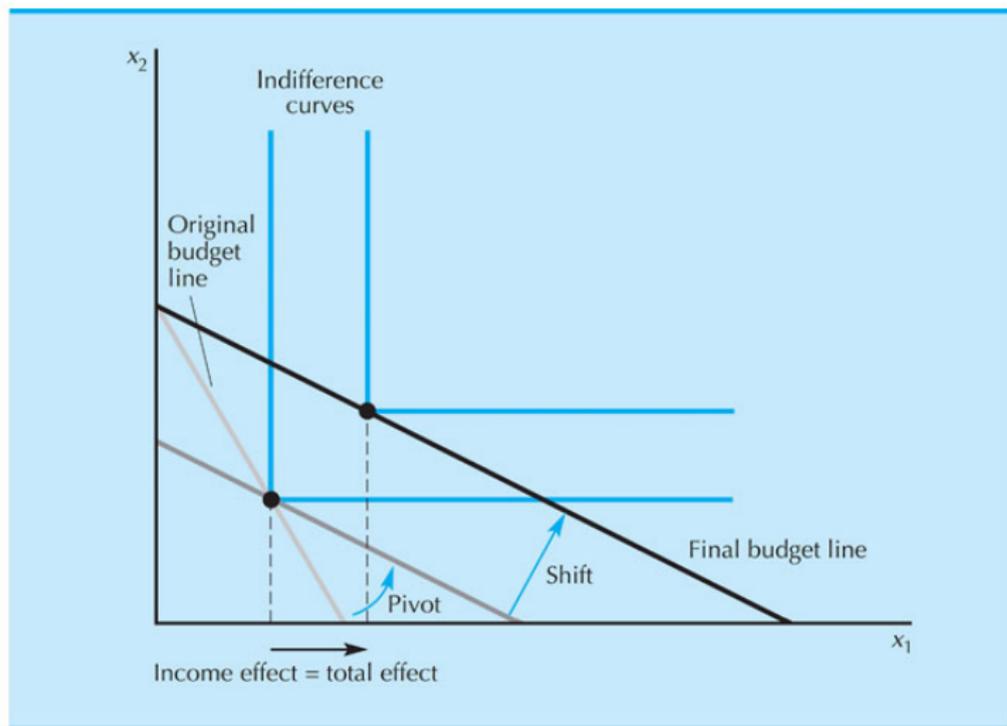


Figure 8.4

# Perfect Substitutes

- ▶ Consider perfect substitutes (starting where all consumption is of good 2)
- ▶ What are the income and substitution effects of a decrease in  $p_1$ ?
  - ▶ Note that after pivot, no shift is required to get back to original income level
  - ▶ Thus income effect is zero
  - ▶ Thus all of demand change (if any) is driven by substitution effect
  - ▶ If change in price is small enough, substitution effect will be zero too

# Perfect Substitutes Graphically

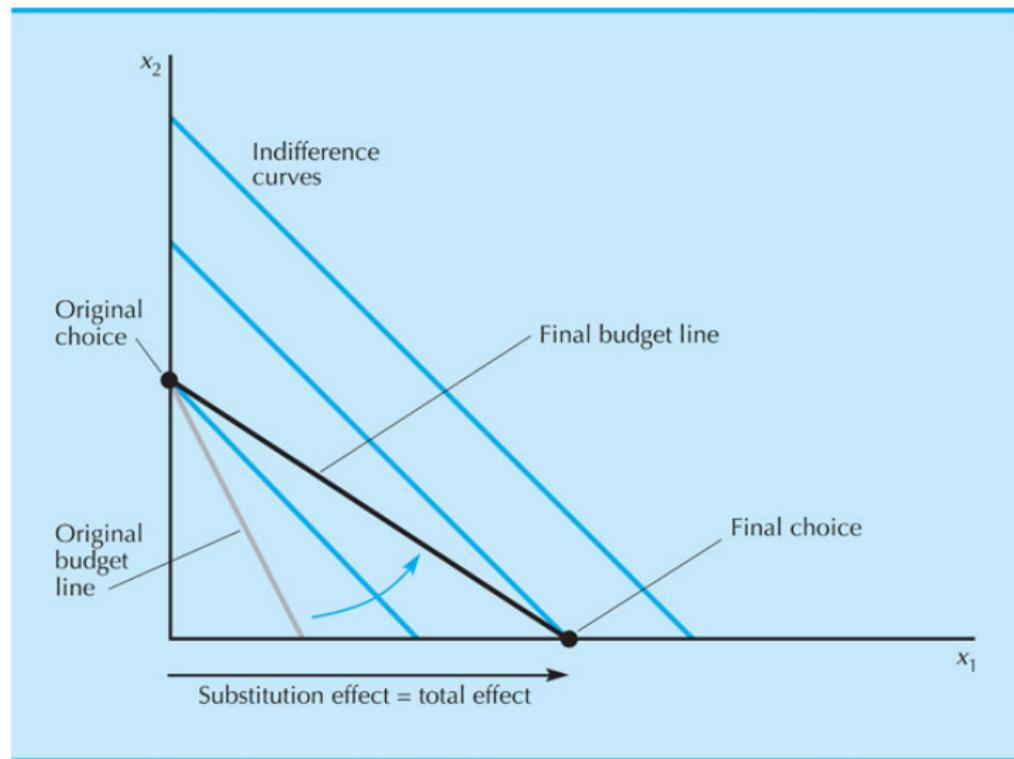


Figure 8.5

# Quasilinear Preferences

- ▶ Quasilinear preferences (quasilinear in good 2):

$$u(x_1, x_2) = v(x_1) + x_2$$

- ▶ Note that MRS depends only on  $x_1$ :

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -v'(x_1)$$

- ▶ Thus the indifference curves are vertical translations of one another
- ▶ Substitution effect: pivot moves consumer to higher indifference curve
- ▶ Income effect: shift moves consumer to tangency point directly above, hence effect is zero

# Quasilinear Preferences Graphically

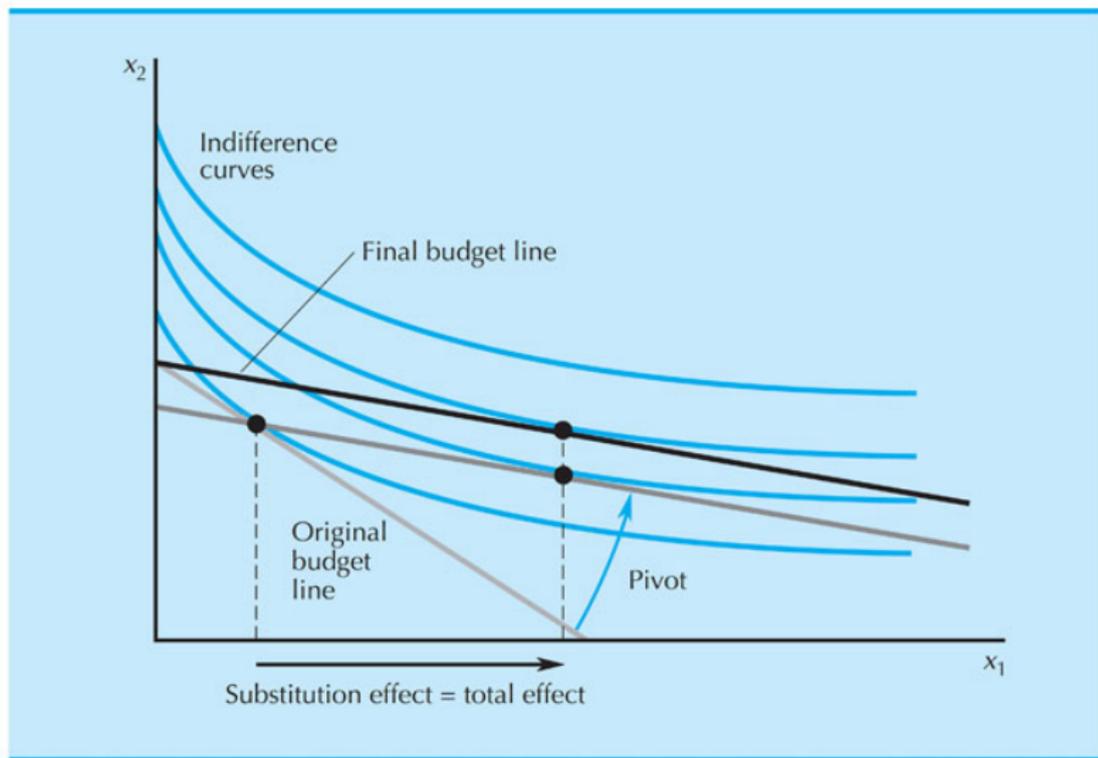


Figure 8.6

# Rates of Change

- ▶ We can make a second formulation of the Slutsky equation
- ▶ First, define the negative income effect as  $\Delta x_1^m = -\Delta x_1^n$
- ▶ Then the Slutsky equation is

$$\Delta x_1 = \Delta x_1^s - \Delta x_1^m$$

- ▶ Divide through by  $\Delta p_1$ :

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}$$

- ▶ Finally, substitute  $\Delta p_1 = \frac{\Delta m}{x_1}$  into rightmost term:

$$\underbrace{\frac{\Delta x_1}{\Delta p_1}}_{\text{total effect}} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{\text{sub effect}} - \underbrace{\frac{\Delta x_1^m}{\Delta m}}_{\text{inc effect}} x_1$$

# Confirming with Example

- ▶ Consider our example from earlier:
  - ▶ Demand function  $x_1 = 10 + \frac{m}{10p_1}$
  - ▶  $p_1 = 3$  and  $m = 120$
  - ▶ Price decrease to  $p'_1 = 2$
- ▶ Does the rates of change version of Slutsky hold?
- ▶ We found:
  - ▶  $x_1 = 14$
  - ▶  $\Delta m = -14$
  - ▶  $\Delta x_1^S = 1.3$
  - ▶  $\Delta x_1^N = 0.7$
- ▶ Confirm Slutsky:
  - ▶  $\frac{\Delta x_1}{\Delta p_1} = \frac{2}{-1} = -2$
  - ▶  $\frac{\Delta x_1^S}{\Delta p_1} + \frac{\Delta x_1^m}{\Delta m} x_1 = \frac{1.3}{-1} - \frac{-0.7}{-14} 14 = -1.3 - 0.7 = -2$

# Slutsky with Calculus

- ▶ We can get a third and final version of Slutsky from calculus principles
- ▶ First, define the *Slutsky demand* function as

$$x_1^S(p_1, p_2, \bar{x}_1, \bar{x}_2) = x_1(p_1, p_2, \underbrace{p_1\bar{x}_1 + p_2\bar{x}_2}_m)$$

where  $(\bar{x}_1, \bar{x}_2)$  is original demand bundle

- ▶ Differentiating:

$$\frac{\partial x_1^S}{\partial p_1} = \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial m} \frac{\partial m}{\partial p_1}$$

- ▶ Finally, noting that  $\frac{\partial m}{\partial p_1} = \bar{x}_1$  and rearranging:

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^S}{\partial p_1} - \frac{\partial x_1}{\partial m} \bar{x}_1$$

# Confirming with Example

▶ Consider our example demand function  $x_1 = 10 + \frac{m}{10p_1}$

▶ Confirm Slutsky:

▶  $\frac{\partial x_1}{\partial p_1} = -\frac{m}{10p_1^2}$

▶ Note that  $x_1^s = 10 + \frac{p_1 x_1 + p_2 x_2}{10p_1} = 10 + \frac{x_1}{10} + \frac{p_2 x_2}{10p_1}$

▶ So  $\frac{\partial x_1^s}{\partial p_1} = -\frac{p_2 x_2}{10p_1^2}$

▶ Note  $p_2 x_2 = m - p_1 x_1$ , so  $\frac{\partial x_1^s}{\partial p_1} = -\frac{m - p_1 x_1}{10p_1^2} = -\frac{m}{10p_1^2} + \frac{x_1}{10p_1}$

▶  $\frac{\partial x_1}{\partial m} = \frac{1}{10p_1}$

▶ Thus  $\frac{\partial x_1^s}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1 = -\frac{m}{10p_1^2} - \frac{x_1}{10p_1} + \frac{1}{10p_1} x_1 = -\frac{m}{10p_1^2}$

## Appendix

# Compensated Demand

- ▶ We can decompose demand change in response to price change in another way
- ▶ First, “roll” budget curve along indifference curve until get to new budget curve slope
  - ▶ This is called *Hicksian demand* or *compensated demand*
  - ▶ Note that we keep utility the same during first move
- ▶ Then, shift demand out by increasing income

# Compensated Demand Decomposition Graphically

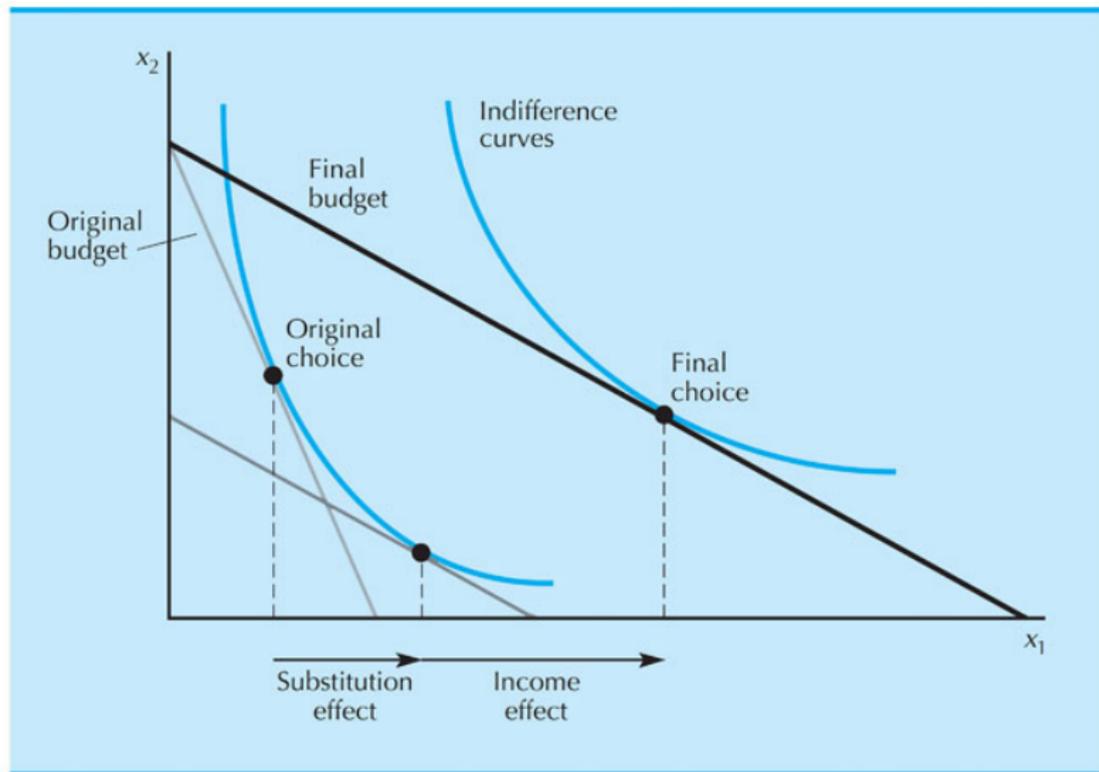


Figure 8.9

# Sign of Compensated Demand Substitution Effect

- ▶ What sign does compensated demand substitution effect (ie “roll”) take?
- ▶ Note that  $(x_1, x_2) \sim (y_1, y_2)$ , so we must have

$$p_1 x_1 + p_2 x_2 \leq p_1 y_1 + p_2 y_2$$

$$p'_1 y_1 + p_2 y_2 \leq p'_1 x_1 + p_2 x_2$$

- ▶ Adding these equations together:

$$(p'_1 - p_1)(y_1 - x_1) + (p_2 - p_2)(y_2 - x_2) \leq 0$$

- ▶ Since second term is zero we get

$$\Delta p_1 \Delta x_1 \leq 0$$

- ▶ Thus a decrease in  $p_1$  causes an increase in compensated demand (just like with Slutsky)

# Different Demands

- ▶ In fact, have a Slutsky-like decomposition using compensated demand:

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^c}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1$$

where  $x_1^c(p_1, p_2, \bar{u})$  is compensated demand for a particular utility level  $\bar{u}$

- ▶ If we want to see how demand changes with price changing and ...
  - ▶ income fixed: use standard demand (also called *Marshallian demand*)
  - ▶ purchasing power fixed: use Slutsky demand
  - ▶ utility fixed: use Hicksian/compensated demand