Econ 301: Microeconomic Analysis

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Expected Utility

Motivating Example: Insurance

- Income is $35,000
- With probability $p = .01$, lose $10,000 to a house fire
- Can buy $10,000 of insurance coverage for $100
  - Then net income will be $34,900 regardless of whether house fire happens or not
- Which option would you rather have?
  1. 99% chance of $35,000 with 1% chance of $25,000
  2. $34,900 for sure
- Consumer will pick option with higher *expected utility*

Contingent Consumption

- Different *states of the world* with corresponding probabilities
- *Contingent consumption plan*: what consumption will be in each state of the world
- For insurance example:
  - Two states of the world: good (no fire) and bad (fire)
  - Bad state occurs with probability π
  - Income M received in either state
  - Loss L if bad state
  - Choice amount of insurance coverage K
  - Insurance premium γ: cost of getting $1 of coverage
  - Contigent consumption plan:

Budget Constraint

- Think of consumption in state 1 as a good and consumption in state 2 as another good
- What is formula for budget constraint?

- What is slope?

- What point does budget line go through?

Budget Constraint Graphically

Expected Utility

- Consider a general contingent consumption plan

$$A = (\pi_i, c_i)_{i=1}^N = (\pi_1, c_1; \pi_2, c_2; \ldots; \pi_N, c_N)$$

  meaning
  - consume c_1 in state 1, which occurs with probability π_1
  - consume c_2 in state 2, which occurs with probability π_2
  - and so on
- A is also called a *gamble*
- The *expected utility* of A is

$$EU(A) = \sum_i \pi_i u(c_i) = \pi_1 u(c_1) + \pi_2 u(c_2) + \ldots + \pi_N u(c_N)$$

- Compare to the *expected value* of A:

$$EV(A) = \sum_i \pi_i c_i = \pi_1 c_1 + \pi_2 c_2 + \ldots + \pi_N c_N$$

Why this formula?

What Shape Should $u(x)$ Have?

- Consider the following game: I will flip a coin until the first heads comes up. If the first heads is on flip number n, then I'll pay you $\$2^n$. How much would you pay to play this game?
  - Originally proposed by Bernoulli (1738, reprinted 1954)
- What is the expected payoff of this game?

- It is clear that there must be *diminishing marginal utility of money*
  - Intuition: an extra \$1000 is massive windfall for a very poor person but not even noticeable for very rich person
- We can rationalize the typically observed behavior by assuming that $u(x)$ is concave

Risk Aversion

- If $u(x)$ is concave, we say the underlying preferences are *risk averse*
  - Recall concavity of u means $u'' < 0$
- If risk averse, then $EU(A) < u(EV(A))$ because of concavity of $u(\cdot)$
  - In words: expected utility of a gamble is less than the utility of its expected value
- Useful tip for drawing EU: If gamble A pays off either x_1 or x_2, then $EU(A)$ lies on the line connecting $u(x_1)$ and $u(x_2)$, directly above $EV(A)$

Risk Aversion Graphically

Certainty Equivalent and Risk Premium

- The *certainty equivalent* of a gamble A is the amount CE such that $u(CE) = EU(A)$
  - That is, certain amount that gives same utility as uncertain gamble
  - How does certainty equivalent relate to expected value of gamble?
- The *risk premium* is the amount $RP = EV(A) - CE$
  - That is, difference between expected value of gamble and certainty equivalent of gamble
  - What is sign of risk premium?

Risk Aversion vs Risk Seeking

- Can also have *risk-seeking* preferences (convex $u(x)$) where all of the above statements are reversed
- Can also have *risk-neutral* preferences (linear $u(x)$)

In summary:

Risk Averse	**Risk Neutral**	**Risk Seeking**
$u(x)$ concave	$u(x)$ linear	$u(x)$ convex
$EU(A) < u(EV(A))$	$EU(A) = u(EV(A))$	$EU(A) > u(EV(A))$
$CE < EV(A)$	$CE = EV(A)$	$CE > EV(A)$
$RP > 0$	$RP = 0$	$RP < 0$

Risk Aversion: Example

- Consider a coin flip for \$15 or \$5
- Let $u(x) = \sqrt{x}$
- Expected value:
- Utility of getting expected value for certain:
- Expected utility of gamble:
- Certainty equivalent of gamble:
- Risk premium of gamble:

Absolute Risk Aversion

- Suppose we want to compare risk aversion across people
- Naively, we may just compare the curvature $u''(x)$
- But this depends on on the scale of utility
- Instead, use the coefficient of *absolute risk aversion*, $-\frac{u''(x)}{u'(x)}$
  - Also know as *Arrow-Pratt measure of risk aversion*
- For risk-averse individual, coefficient must be positive
- Person with higher coefficient is more risk averse

Interpreting Absolute Risk Aversion

- Coefficient may be constant, increasing, or decreasing as x increases
- Constant absolute risk aversion (CARA): as wealth increases, hold same number of dollars in risky asset
- Increasing absolute risk aversion (IARA): as wealth increases, hold fewer dollars in risky asset
- Decreasing absolute risk aversion (DARA): as wealth increases, hold more dollars in risky asset

Relative Risk Aversion

- May want to scale by wealth/income x
- Use *coefficient of relative risk aversion*, $-x\frac{u''(x)}{u'(x)}$
- For risk-averse individual, coefficient must be positive (for positive x)
- Coefficient may be constant (CRRA), increasing (IRRA), or decreasing (DRRA) as x increases
  - Constant relative risk aversion (CRRA): as wealth increases, hold same percentage of dollars in safe asset
  - Increasing relative risk aversion (IRRA): as wealth increases, hold higher percentage of dollars in safe asset
  - Decreasing relative risk aversion (DRRA): as wealth increases, hold lower percentage of dollars in safe asset

Examples

- Does utility function $u(x) = \ln(x)$ exhibit increasing, decreasing, or constant absolute risk aversion?
- Does utility function $u(x) = \sqrt{x}$ exhibit increasing, decreasing, or constant absolute risk aversion?
- Which one is more risk averse?

Returning to the Insurance Example

- What is expected profit for insurance company?
- What should γ equal in a competitive insurance market?

Consumer Behavior Under Fair Insurance

- What is expected utility of consumer as function of K? (Assume they have utility function u)
- What is optimal insurance coverage K?

Appendix

Why Is Expected Utility Reasonable?

- Suppose you make just a few innocuous assumptions about preferences between gambles:
  1. Completeness: For any gambles A and B, either $A \succeq B$ or $B \succeq A$ (or both).
  2. Transitivity: For any gambles A, B, and C, if $A \succeq B$ and $B \succeq C$, then $A \succeq C$.
  3. Continuity: For any gambles A, B, and C, if $A \succeq B \succeq C$ then there exists some number $p \in (0, 1]$ such that $pA + (1 - p)C \sim B$.
  4. Independence: For any gambles A, B, C such that $A \succeq B$ and any $p \in (0, 1]$, we must have $pA + (1 - p)C \succeq pB + (1 - p)C$.

Theorem (von Nueman and Moregensten)

Preferences over gambles that satisfy conditions 1-4 can be represented by expected utility.

The Importance of Independence

- The independence axiom is the most important one for expected utility theory
- What is the intuition for this axiom?
  - How you feel about a prize (ie a specific amount of money) does not depend on the probability you receive it
- How does this manifest in EUT?
  - Let gamble A have three possible outcomes, i.e. $A = (\pi_1, x_1; \pi_2, x_2; \pi_3, x_3)$
  - Recall $EU(A) = \sum_i \pi_i u(x_i) = \pi_1 u(x_1) + \pi_2 u(x_2) + \pi_3 u(x_3)$
  - Note that utility is *additively separable* in probabilities
    - That is, $EU(\pi_1, \pi_2, \pi_3, \cdot) = f_1(\pi_1) + f_1(\pi_2) + f_3(\pi_3)$
  - Note that utility is *linear* in probabilities
    - That is, $EU(\pi_i, \cdot) = a\pi_i + b$ for some constants a and b

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