Econ 301: Microeconomic Analysis

Prof. Jeffrey Naecker

Wesleyan University

The Slutsky Equation

Total Change in Demand

- Last time we defined
  - Income effect: $\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m')$
  - Substitution effect: $\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m)$
- Note that the total change in demand is

$$\begin{aligned}
\Delta x_1 &= x_1(p_1', m) - x_1(p_1, m) \\
&= x_1(p_1', m') - x_1(p_1, m) + x_1(p_1', m) - x_1(p_1', m') \\
&= \Delta x_1^s + \Delta x_1^n
\end{aligned}$$

- This is one form of the *Slutsky identity* or *Slutsky equation*

Signing the Total Change in Demand

- What sign does the overall change in demand take if $\Delta p_1 < 0$?

Signing the Total Change in Demand

- What sign does the overall change in demand take if $\Delta p_1 < 0$?
- We showed that $\Delta x_1^s > 0$ if $\Delta p_1 < 0$
- What if good 1 is normal?

Signing the Total Change in Demand

- What sign does the overall change in demand take if $\Delta p_1 < 0$?
- We showed that $\Delta x_1^s > 0$ if $\Delta p_1 < 0$
- What if good 1 is normal?
  - Then $\Delta x_1^n > 0, so \Delta x_1 > 0$

Signing the Total Change in Demand

- What sign does the overall change in demand take if $\Delta p_1 < 0$?
- We showed that $\Delta x_1^s > 0$ if $\Delta p_1 < 0$
- What if good 1 is normal?
  - Then $\Delta x_1^n > 0, so \Delta x_1 > 0$
- What if good 1 is inferior?

Signing the Total Change in Demand

- What sign does the overall change in demand take if $\Delta p_1 < 0$?
- We showed that $\Delta x_1^s > 0$ if $\Delta p_1 < 0$
- What if good 1 is normal?
  - Then $\Delta x_1^n > 0, so \Delta x_1 > 0$
- What if good 1 is inferior?
  - Then $\Delta x_1^n < 0$, so sign of Δx_1 is ambiguous
  - Note $\Delta x_1 < 0$ (a Giffen good) if $|\Delta x_1^n| > |\Delta x_1^s|$

Signing the Total Change in Demand

- What sign does the overall change in demand take if $\Delta p_1 < 0$?
- We showed that $\Delta x_1^s > 0$ if $\Delta p_1 < 0$
- What if good 1 is normal?
  - Then $\Delta x_1^n > 0, so \Delta x_1 > 0$
- What if good 1 is inferior?
  - Then $\Delta x_1^n < 0$, so sign of Δx_1 is ambiguous
  - Note $\Delta x_1 < 0$ (a Giffen good) if $|\Delta x_1^n| > |\Delta x_1^s|$
- Are all Giffen goods inferior?

Signing the Total Change in Demand

- What sign does the overall change in demand take if $\Delta p_1 < 0$?
- We showed that $\Delta x_1^s > 0$ if $\Delta p_1 < 0$
- What if good 1 is normal?
  - Then $\Delta x_1^n > 0, so \Delta x_1 > 0$
- What if good 1 is inferior?
  - Then $\Delta x_1^n < 0$, so sign of Δx_1 is ambiguous
  - Note $\Delta x_1 < 0$ (a Giffen good) if $|\Delta x_1^n| > |\Delta x_1^s|$
- Are all Giffen goods inferior? Yes
- Are all inferior goods Giffen?

Signing the Total Change in Demand

- What sign does the overall change in demand take if $\Delta p_1 < 0$?
- We showed that $\Delta x_1^s > 0$ if $\Delta p_1 < 0$
- What if good 1 is normal?
  - Then $\Delta x_1^n > 0, so \Delta x_1 > 0$
- What if good 1 is inferior?
  - Then $\Delta x_1^n < 0$, so sign of Δx_1 is ambiguous
  - Note $\Delta x_1 < 0$ (a Giffen good) if $|\Delta x_1^n| > |\Delta x_1^s|$
- Are all Giffen goods inferior? Yes
- Are all inferior goods Giffen? No

Giffen and Inferior Goods Graphically

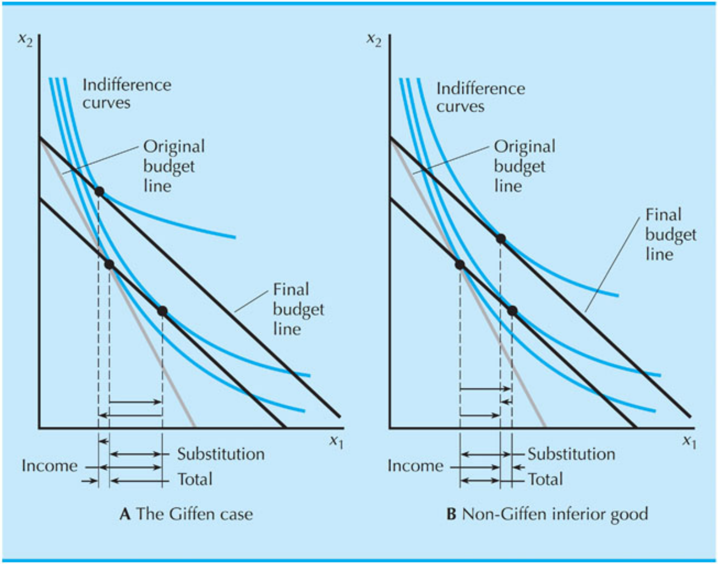

Figure 8.3

Example

- Let demand function be given by $x_1 = 10 + \frac{m}{10p_1}$
- Suppose we start out at $p_1 = 3$ and $m = 120$
- Suppose price decrease to $p_1' = 2$
- Substitution effect?

Example

- Let demand function be given by $x_1 = 10 + \frac{m}{10p_1}$
- Suppose we start out at $p_1 = 3$ and $m = 120$
- Suppose price decrease to $p_1' = 2$
- Substitution effect?
  - Starting demand $x_1 =$

Example

- Let demand function be given by $x_1 = 10 + \frac{m}{10p_1}$
- Suppose we start out at $p_1 = 3$ and $m = 120$
- Suppose price decrease to $p_1' = 2$
- Substitution effect?
  - Starting demand $x_1 = 10 + \frac{120}{30} = 14$

Example

- Let demand function be given by $x_1 = 10 + \frac{m}{10p_1}$
- Suppose we start out at $p_1 = 3$ and $m = 120$
- Suppose price decrease to $p_1' = 2$
- Substitution effect?
  - Starting demand $x_1 = 10 + \frac{120}{30} = 14$
  - $\Delta m = x_1 \Delta p_1 = 14(-1) = -14$
  - $m' = 120 - 14 = 106$
  - Intermediate demand $x_1(p_1', m') = 10 + \frac{106}{20} = 15.3$
  - $\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m) = 15.3 - 14 = 1.3$
- Income effect?

Example

- Let demand function be given by $x_1 = 10 + \frac{m}{10p_1}$
- Suppose we start out at $p_1 = 3$ and $m = 120$
- Suppose price decrease to $p_1' = 2$
- Substitution effect?
  - Starting demand $x_1 = 10 + \frac{120}{30} = 14$
  - $\Delta m = x_1 \Delta p_1 = 14(-1) = -14$
  - $m' = 120 - 14 = 106$
  - Intermediate demand $x_1(p_1', m') = 10 + \frac{106}{20} = 15.3$
  - $\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m) = 15.3 - 14 = 1.3$
- Income effect?
  - Final demand $x_1(p_1', m) = 10 + \frac{120}{20} = 16$
  - $\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m') = 16 - 15.3 = 0.7$

Perfect Complements

- Consider perfect complements preferences
- What are the income and substitution effects of a decrease in p_1?

Perfect Complements

- Consider perfect complements preferences
- What are the income and substitution effects of a decrease in p_1?
  - Note that pivot has no effect on optimal consumption
  - Thus substitution effect is zero
  - Thus demand change is entirely from income effect

Perfect Complements Graphically

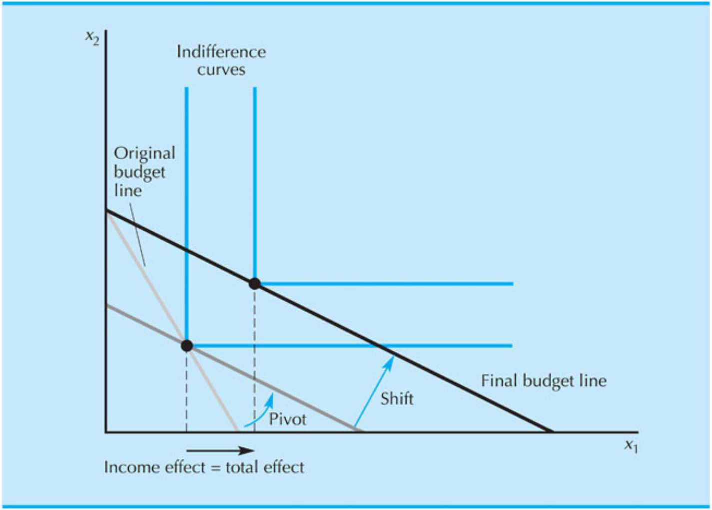

Figure 8.4

Perfect Substitutes

- Consider perfect substitutes (starting where all consumption is of good 2)
- What are the income and substitution effects of a decrease in p_1?

Perfect Substitutes

- Consider perfect substitutes (starting where all consumption is of good 2)
- What are the income and substitution effects of a decrease in p_1?
  - Note that after pivot, no shift is required to get back to original income level
  - Thus income effect is zero
  - Thus all of demand change (if any) is driven by substitution effect
  - If change in price is small enough, substitution effect wills be zero too

Perfect Substitutes Graphically

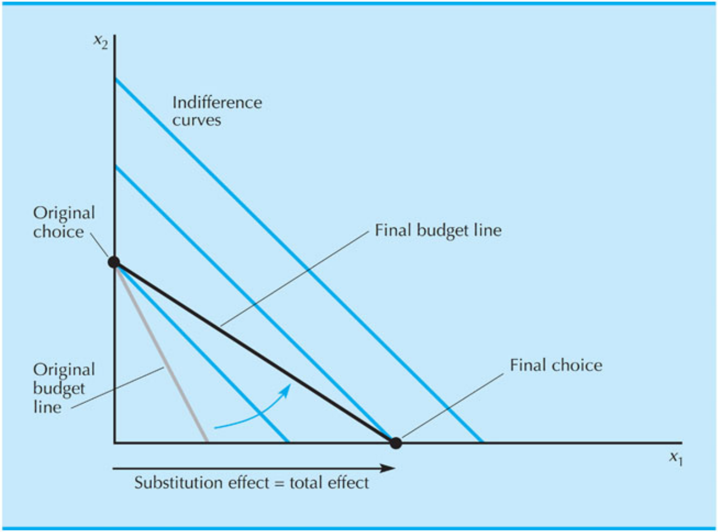

Figure 8.5

Quasilinear Preferences

- Quasilinear preferences (quasilinear in good 2):

$$u(x_1, x_2) = v(x_1) + x_2$$

- Note that MRS depends only on x_1:

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -v'(x_1)$$

- Thus the indifference curves are vertical translations of one another

Quasilinear Preferences

- Quasilinear preferences (quasilinear in good 2):

$$u(x_1, x_2) = v(x_1) + x_2$$

- Note that MRS depends only on x_1:

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -v'(x_1)$$

- Thus the indifference curves are vertical translations of one another
- Substitution effect:

Quasilinear Preferences

- Quasilinear preferences (quasilinear in good 2):

$$u(x_1, x_2) = v(x_1) + x_2$$

- Note that MRS depends only on x_1:

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -v'(x_1)$$

- Thus the indifference curves are vertical translations of one another
- Substitution effect: pivot moves consumer to higher indifference curve

Quasilinear Preferences

- Quasilinear preferences (quasilinear in good 2):

$$u(x_1, x_2) = v(x_1) + x_2$$

- Note that MRS depends only on x_1:

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -v'(x_1)$$

- Thus the indifference curves are vertical translations of one another
- Substitution effect: pivot moves consumer to higher indifference curve
- Income effect:

Quasilinear Preferences

- Quasilinear preferences (quasilinear in good 2):

$$u(x_1, x_2) = v(x_1) + x_2$$

- Note that MRS depends only on x_1:

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -v'(x_1)$$

- Thus the indifference curves are vertical translations of one another
- Substitution effect: pivot moves consumer to higher indifference curve
- Income effect: shift moves consumer to tangency point directly above, hence effect is zero

Quasilinear Preferences Graphically

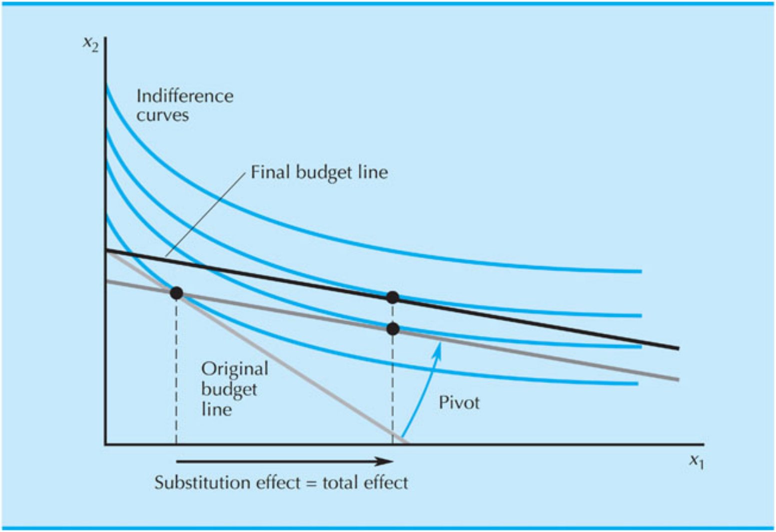

Figure 8.6

Rates of Change

- We can make a second formulation of the Slutsky equation
- First, define the negative income effect as $\Delta x_1^m = -\Delta x_1^n$
- Then the Slutsky equation is

$$\Delta x_1 = \Delta x_1^s - \Delta x_1^m$$

Rates of Change

- We can make a second formulation of the Slutsky equation
- First, define the negative income effect as $\Delta x_1^m = -\Delta x_1^n$
- Then the Slutsky equation is

$$\Delta x_1 = \Delta x_1^s - \Delta x_1^m$$

- Divide through by Δp_1:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}$$

Rates of Change

- We can make a second formulation of the Slutsky equation
- First, define the negative income effect as $\Delta x_1^m = -\Delta x_1^n$
- Then the Slutsky equation is

$$\Delta x_1 = \Delta x_1^s - \Delta x_1^m$$

- Divide through by Δp_1:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}$$

- Finally, substitute $\Delta p_1 = \frac{\Delta m}{x_1}$ into rightmost term:

$$\underbrace{\frac{\Delta x_1}{\Delta p_1}}_{\text{total effect}} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{\text{sub effect}} - \underbrace{\frac{\Delta x_1^m}{\Delta m}}_{\text{inc effect}} x_1$$

Confirming with Example

- Consider our example from earlier:
  - Demand function $x_1 = 10 + \frac{m}{10p_1}$
  - $p_1 = 3$ and $m = 120$
  - Price decrease to $p_1' = 2$
- Does the rates of change version of Slutsky hold?

Confirming with Example

- Consider our example from earlier:
  - Demand function $x_1 = 10 + \frac{m}{10p_1}$
  - $p_1 = 3$ and $m = 120$
  - Price decrease to $p_1' = 2$
- Does the rates of change version of Slutsky hold?
- We found:
  - $x_1 = 14$
  - $\Delta m = -14$
  - $\Delta x_1^s = 1.3$
  - $\Delta x_1^n = 0.7$

Confirming with Example

- Consider our example from earlier:
  - Demand function $x_1 = 10 + \frac{m}{10p_1}$
  - $p_1 = 3$ and $m = 120$
  - Price decrease to $p_1' = 2$
- Does the rates of change version of Slutsky hold?
- We found:
  - $x_1 = 14$
  - $\Delta m = -14$
  - $\Delta x_1^s = 1.3$
  - $\Delta x_1^n = 0.7$
- Confirm Slutsky:
  - $\frac{\Delta x_1}{\Delta p_1} = \frac{2}{-1} = -2$
  - $\frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^m}{\Delta m} x_1 = \frac{1.3}{-1} - \frac{-0.7}{-14} 14 = -1.3 - 0.7 = -2$

Slutsky with Calculus

- We can get a third and final version of Slutsky from calculus principles
- First, define the *Slutsky demand* function as

$$x_1^s(p_1, p_2, \bar{x}_1, \bar{x}_2) = x_1(p_1, p_2, \underbrace{p_1\bar{x}_1 + p_2\bar{x}_2}_{m})$$

where $(\bar{x}_1, \bar{x}_2)$ is original demand bundle

Slutsky with Calculus

- We can get a third and final version of Slutsky from calculus principles
- First, define the *Slutsky demand* function as

$$x_1^s(p_1, p_2, \bar{x}_1, \bar{x}_2) = x_1(p_1, p_2, \underbrace{p_1\bar{x}_1 + p_2\bar{x}_2}_{m})$$

  where $(\bar{x}_1, \bar{x}_2)$ is original demand bundle
- Differentiating:

$$\frac{\partial x_1^s}{\partial p_1} = \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial m}\frac{\partial m}{\partial p_1}$$

Slutsky with Calculus

- We can get a third and final version of Slutsky from calculus principles
- First, define the *Slutsky demand* function as

$$x_1^s(p_1, p_2, \bar{x}_1, \bar{x}_2) = x_1(p_1, p_2, \underbrace{p_1\bar{x}_1 + p_2\bar{x}_2}_{m})$$

  where $(\bar{x}_1, \bar{x}_2)$ is original demand bundle
- Differentiating:

$$\frac{\partial x_1^s}{\partial p_1} = \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial m}\frac{\partial m}{\partial p_1}$$

- Finally, noting that $\frac{\partial m}{\partial p_1} = \bar{x}_1$ and rearranging:

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^s}{\partial p_1} - \frac{\partial x_1}{\partial m}\bar{x}_1$$

Confirming with Example

- Consider our example demand function $x_1 = 10 + \frac{m}{10p_1}$

Confirming with Example

- Consider our example demand function $x_1 = 10 + \frac{m}{10p_1}$
- Confirm Slutsky:
  - $\frac{\partial x_1}{\partial p_1} = -\frac{m}{10p_1^2}$

Confirming with Example

- Consider our example demand function $x_1 = 10 + \frac{m}{10p_1}$
- Confirm Slutsky:
  - $\frac{\partial x_1}{\partial p_1} = -\frac{m}{10p_1^2}$
  - Note that $x_1^s = 10 + \frac{p_1 x_1 + p_2 x_2}{10p_1} = 10 + \frac{x_1}{10} + \frac{p_2 x_2}{10p_1}$
    - So $\frac{\partial x_1^s}{\partial p_1} = -\frac{p_2 x_2}{10p_1^2}$
    - Note $p_2 x_2 = m - p_1 x_1$, so $\frac{\partial x_1^s}{\partial p_1} = -\frac{m - p_1 x_1}{10p_1^2} = -\frac{m}{10p_1^2} + \frac{x_1}{10p_1}$

Confirming with Example

- Consider our example demand function $x_1 = 10 + \frac{m}{10p_1}$
- Confirm Slutsky:
  - $\frac{\partial x_1}{\partial p_1} = -\frac{m}{10p_1^2}$
  - Note that $x_1^s = 10 + \frac{p_1 x_1 + p_2 x_2}{10p_1} = 10 + \frac{x_1}{10} + \frac{p_2 x_2}{10p_1}$
    - So $\frac{\partial x_1^s}{\partial p_1} = -\frac{p_2 x_2}{10p_1^2}$
    - Note $p_2 x_2 = m - p_1 x_1$, so $\frac{\partial x_1^s}{\partial p_1} = -\frac{m - p_1 x_1}{10p_1^2} = -\frac{m}{10p_1^2} + \frac{x_1}{10p_1}$
  - $\frac{\partial x_1}{\partial m} = \frac{1}{10p_1}$

Confirming with Example

- Consider our example demand function $x_1 = 10 + \frac{m}{10p_1}$
- Confirm Slutsky:
  - $\frac{\partial x_1}{\partial p_1} = -\frac{m}{10p_1^2}$
  - Note that $x_1^s = 10 + \frac{p_1 x_1 + p_2 x_2}{10p_1} = 10 + \frac{x_1}{10} + \frac{p_2 x_2}{10p_1}$
    - So $\frac{\partial x_1^s}{\partial p_1} = -\frac{p_2 x_2}{10p_1^2}$
    - Note $p_2 x_2 = m - p_1 x_1$, so $\frac{\partial x_1^s}{\partial p_1} = -\frac{m - p_1 x_1}{10p_1^2} = -\frac{m}{10p_1^2} + \frac{x_1}{10p_1}$
  - $\frac{\partial x_1}{\partial m} = \frac{1}{10p_1}$
  - Thus $\frac{\partial x_1^s}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1 = -\frac{m}{10p_1^2} - \frac{x_1}{10p_1} + \frac{1}{10p_1} x_1 = -\frac{m}{10p_1^2}$

Appendix

Compensated Demand

- We can decompose demand change in response to price chance in another way
- First, “roll” budget curve along indifference curve until get to new budget curve slope
  - This is called *Hicksian demand* or *compensated demand*
  - Note that we keep utility the same during first move
- Then, shift demand out by increasing income

Compensated Demand Decomposition Graphically

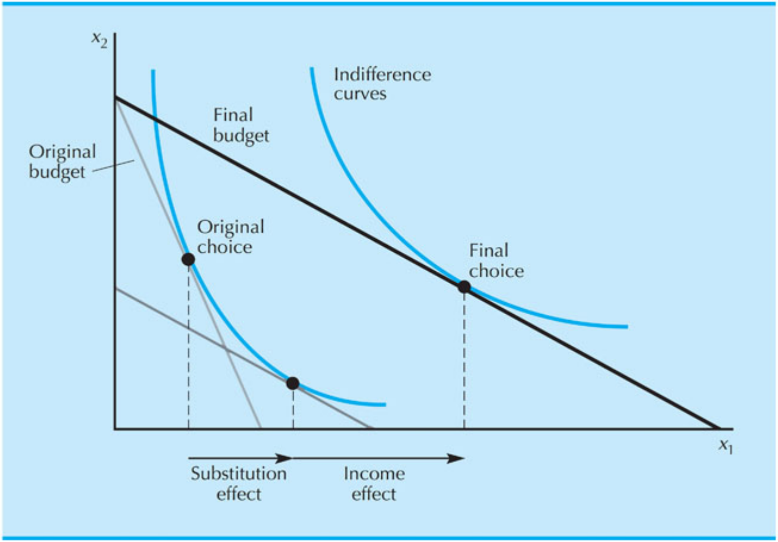

Figure 8.9

Sign of Compensated Demand Substitution Effect

- What sign does compensated demand substitution effect (ie “roll”) take?

Sign of Compensated Demand Substitution Effect

- What sign does compensated demand substitution effect (ie “roll”) take?
- Note that $(x_1, x_2) \sim (y_1, y_2)$, so we must have

$$p_1 x_1 + p_2 x_2 \leq p_1 y_1 + p_2 y_2$$
$$p_1' y_1 + p_2 y_2 \leq p_1' x_1 + p_2 x_2$$

- Adding these equations together:

$$(p_1' - p_1)(y_1 - x_1) + (p_2 - p_2)(y_2 - x_2) \leq 0$$

- Since second term is zero we get

$$\Delta p_1 \Delta x_1 \leq 0$$

- Thus a decrease in p_1 causes an increase in compensated demand (just like with Slutsky)

Different Demands

- In fact, have a Slutsky-like decomposition using compensated emand:
$$\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^c}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1$$
where $x_1^c(p_1, p_2, \bar{u})$ is compensated demand for a particular utility level $\bar{u}$

Different Demands

- In fact, have a Slutsky-like decomposition using compensated emand:

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^c}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1$$

  where $x_1^c(p_1, p_2, \bar{u})$ is compensated demand for a particular utility level $\bar{u}$
- If we want to see how demand changes with price changing and …
  - income fixed: use standard demand (also called *Marshallian demand*)

Different Demands

- In fact, have a Slutsky-like decomposition using compensated emand:
$$\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^c}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1$$
where $x_1^c(p_1, p_2, \bar{u})$ is compensated demand for a particular utility level $\bar{u}$
- If we want to see how demand changes with price changing and ...
  - income fixed: use standard demand (also called *Marshallian demand*)
  - purchasing power fixed: use Slutsky demand

Different Demands

- In fact, have a Slutsky-like decomposition using compensated emand:

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^c}{\partial p_1} - \frac{\partial x_1}{\partial m} x_1$$

  where $x_1^c(p_1, p_2, \bar{u})$ is compensated demand for a particular utility level $\bar{u}$
- If we want to see how demand changes with price changing and ...
  - income fixed: use standard demand (also called *Marshallian demand*)
  - purchasing power fixed: use Slutsky demand
  - utility fixed: use Hicksian/compensated demand