Econ 301: Microeconomic Analysis

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Course Overview

What This Course is About

- Tools that economists use to analyze behavior
  - Calculus
  - Optimization
  - Equilibrium
  - Game theory
- Applications to that economists typically study
  - Consumers
  - Producers
  - Governments
  - Markets
  - Small groups

Consumers

- How to people choose between options? *Preferences, choices*
- How do we summarize their preferences with a convenient mathematical function? *Utility*
- How do people deal with limited options? *Budget constraints*
- How do people make choices when uncertainty is involved? *Expected utility, asset markets*
- How do people make choices over several time periods? *Intertemporal choice, price indexes*
- How do these choices affect entire markets? *Consumer surplus, market demand*

Producers

- How do firms decide how much to produce? *Technology, cost minimization, cost curves*
- How do these choices affect the entire market? *Firm supply, industry supply*
- What happens if you don't have a competitive market? *Monopoly, oligopoly*

Interactions

- How do supply and demand interact? *Equilibrium*
- What are the side effects of markets? *Externalities*
- What happens if small groups interact rather than large markets? *Game theory, game applications*
- What happens if one side of the market knows more than the other? *Asymmetric information*

Thinking like an Economist

1. Start with a model
   - Agent or agents
   - Their objectives
   - Options available to them
2. Solve the model
   - Optimize behavior of agents, usually with calculus
   - If agents interact, find equilibrium
3. Poke the model
   - What happens if constraints change? *Comparative statics*
   - Could agents be made better off? *Welfare, efficiency*

Who You Are

- Econ majors and minors
- Should have already taken Econ 300
- Introduce yourselves
  - First name
  - Major(s)
  - Where you're from
  - Tell us something we don't know

Administrative Details

- Syllabus overview
- Moodle overview

Mindset

- May not get everything in lecture the first time
  - Review your notes after lecture
  - Rework examples from slides
- Iterate to success:
  - Work by yourself
  - Work with study group
  - Go to TA session
  - Go to office hours
  - Post to Piazza
  - Repeat as needed
- Many PS and exam problems will require a translation or wrinkle of some kind

Advice from Former Students

- “Work hard. Attend classes, pay attention, and interact actively.”
- “Go to every class, and go to extra help sessions whenever possible.”
- “Study the material week by week, not just before a big test. Knowledge is cumulative in this class.”
- “[M]ake a study group to collaborate on problem sets and test review.”
- “Go to the TA. [. . .] Work through as many problems as you can get your hands on. Dont be afraid to ask questions.”
- “Do not be afraid to answer questions when he randomly calls on you, he does not bite.”

Mathematics Review

Mathematics in This Course

- Because this course is about microeconomic ***theory***, deal often with abstract equations and quantities
- Use lots of algebra and calculus
- For reasons outside my control, we will do some multivariate calculus even though this is not a pre-req
- If this is new to you, ask for help from me/TAs/classmates/math workshop

Algebra Tricks

- Exponents:
  $x^a x^b = x^{(a+b)}$
  $\frac{x^a}{x^b} = x^a x^{-b} = x^{(a-b)}$
  $(x^m)^n = x^{mn}$

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- Logarithms:

  $\log x^b = b \log x$
  $\log(xy) = \log x + \log y$
  $\log(\frac{x}{y}) = \log x - \log y$

Basic Derivatives

	Function	Derivative
Derivative of a constant	$f(x) = a$	$\frac{d}{dx}f(x) = 0$
Derivative of a linear function	$f(x) = ax + b$	$\frac{d}{dx}f(x) = a$
Power rule	$f(x) = x^k$	$\frac{d}{dx}f(x) = kx^{k-1}$
Derivative of a constant multiple	$f(x) = ag(x)$	$\frac{d}{dx}f(x) = a\frac{d}{dx}g(x)$
Derivative of a sum	$f(x) = g(x) + h(x)$	$\frac{d}{dx}f(x) = \frac{d}{dx}g(x) + \frac{d}{dx}h(x)$
Product rule	$f(x) = g(x)h(x)$	$\frac{d}{dx}f(x) = g(x)\frac{d}{dx}h(x) + \frac{d}{dx}g(x)h(x)$
Quotient rule	$f(x) = \frac{g(x)}{h(x)}$	$\frac{d}{dx}f(x) = \frac{h(x)\frac{d}{dx}g(x) - \frac{d}{dx}h(x)g(x)}{[h(x)]^2}$
Chain rule	$f(x) = g(h(x))$	$\frac{d}{dx}f(x) = g'(h(x)) \cdot h'(x)$
Derivative of the natural log	$f(x) = \ln(x)$	$\frac{d}{dx}f(x) = \frac{1}{x}$
Derivative of the exponential	$f(x) = e^x$	$\frac{d}{dx}f(x) = e^x$

Notation: $\frac{d}{dx}f(x) = \frac{df}{dx} = f'(x)$

Calculus of Multiple Variables

- Function can have more than one variable: eg, $f(x, y) = x + y$
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- Notation:
  - $\frac{\partial}{\partial x}(f(x,y)) = \frac{\partial f}{\partial x} = f_x(x,y) = f_1(x,y)$
  - $\frac{\partial}{\partial y}(f(x,y)) = \frac{\partial f}{\partial y} = f_y(x,y) = f_2(x,y)$
- We use the notation $\frac{df}{dx}$ to indicate a *total* derivative, where we take into account any variables that vary with x
- Typically (but not always) we are interested in the partial derivative in this class

Total vs Partial Derivative

- For example, suppose $f(x, y) = xy$ but we know that $y = x^2$
  - What is partial derivative of f w.r.t x (as a function of y)?

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- More generally, if $x = x(t)$ and $y = y(t)$, then

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

by the chain rule and the product rule

Integration

- You should also know how to integrate functions of single and multiple variables as well
  - This is much less common than differentiation in this class
- To find the integral of a function $f(x)$, think about what function $F(x) + C$ would have derivative $f(x)$
  - This approach makes use of the fundamental theorem of calculus:

Theorem (FTOC)

Let f be a continuous function on the interval $[a, b]$. If F is the indefinite integral of f, then $\int_a^b f(x)dx = F(b) - F(a) \equiv F(x)|_{x=a}^{x=b}$.

Unconstrained Optimization: One Variable

- Suppose we are trying to find the optimum (ie maximum or minimum) of a function $f(x)$
- We use the *first order condition* to solve for the optimum:

$$\frac{d}{dx}f(x) = 0$$

- Value of x that satisfy the first order condition will be the values that optimizes $f(x)$
- How do we know if we have a maximum or a minimum?
  - We check the *second order conditions*:
    - If $\frac{d^2f}{dx^2} < 0$, we have a maximum
    - If $\frac{d^2f}{dx^2} > 0$, we have a minimum

Unconstrained Optimization: Multiple Variables

- Suppose we are trying to find the optimum of a function $f(x, y)$
- Again, we use the *first order conditions* to solve for the optimum, just now there are more than one:

$$\begin{aligned} \frac{\partial}{\partial x}(f(x, y)) &= 0 \\ \frac{\partial}{\partial y}(f(x, y)) &= 0 \end{aligned}$$

- Values of x and y that satisfy the first order conditions will be the values that optimize $f(x, y)$

Constrained Optimization

- Suppose we are trying to find the optimum of the the function $f(x, y)$ subject to the constraint $b(x, y) \leq m$.
- First construct the *Lagrangian*:

$$\mathcal{L}(x, y, \lambda) = f(x, y) + \lambda(m - b(x, y))$$

- The variable λ is called the *Lagrange multiplier*

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- Take the first order conditions of the Lagrangian:

$$\frac{\partial}{\partial x}(\mathcal{L}(x, y, \lambda)) = 0 \qquad \frac{\partial}{\partial \lambda}(\mathcal{L}(x, y, \lambda)) = 0 \qquad \frac{\partial}{\partial y}(\mathcal{L}(x, y, \lambda)) = 0$$

- Values of x and y that solve the FOC of the Lagrangian will be the same values that optimized the original function f subject to the constraint.