Econ 301: Microeconomic Analysis

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Game Applications

The Prisoner's Dilemma

- Recall Prisoner's Dilemma from last lecture:
  - Two suspects are being interrogated in two separate rooms
  - If they both Deny, go to jail for 2 years
  - If one Confesses, he gets 1 year while other gets 5
  - If they both Confess, go to jail for 4 years

	Deny	*Confess*
Deny	$(-2, -2)$	$(-5, -1)$
Confess	$(-1, -5)$	$(-4, -4)$

- What will happen in this setting?
  - All of our solution concepts agree that (Confess, Confess) is only reasonable outcome

Cooperation in Prisoner's Dilemma

- Is this outcome the most preferred for both players?
  - No, both players would prefer (Deny, Deny) to the Nash equilibrium
- Why doesn't this outcome get played?
  - Players have incentive to double-cross (gain of 1 at cost of 5 for opponent)
- How can we modify game to make this a sustainable outcome?
  - One possibility: repetition
  - Players may attempt to enforce cooperation by threatening with retaliation in future rounds
  - Let's see if this will work

Finitely Repeated Prisoner's Dilemma

- Suppose prisoners are interacting for 10 rounds
- Each round's payoffs are given by the standard one-period game
- What is predicted outcome of the game?
  - Use backwards induction, since this now sequential game
  - What happens in round 10?
    - With no remaining rounds for punishment, players will clearly play (Confess, Confess)
  - Move to round 9:
    - Any threatened punishment is not credible, since we already know what players will do in last round
    - Thus players will play (Confess, Confess) in round 9
  - Round 8:
    - Same logic as round 9, and so on
  - Thus unique backwards induction (or subgame perfect) equilibrium is for both players to play Confess each round
  - So repeating the game does not improve cooperation!

Game Theory with Firms

- Suppose we have a duopoly: just two firms producing in market
- Firms have just two strategies: pricing high or pricing low
- If both price high, split monopoly profits (3 each)
- If both price low, each gets competitive market profit of 2
- If one prices high and one prices low, profits are 1 for high price and 4 for low price firm
- How do we represent this as a game?

	High	*Low*
High	$(3, 3)$	$(1, 4)$
Low	$(4, 1)$	$(2, 2)$

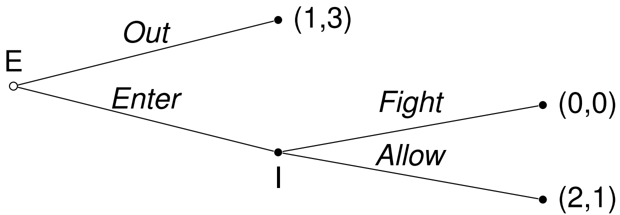
- What is Nash equilbrium of this game?
  - Completely analogous to prisoner's dilemma
  - (Low, Low) is unique Nash equilibrium

Entry Deterrence

- Now suppose we have a monopoly, but a new firm is considering entering market
- Two players: incumbent and entrant
- Entrant chooses to enter or not enter (out)
- If Entrant does enter, monopolist can fight or allow
  - If fight, both firms get payoff 0
  - If allow, payoffs are 2 for entrant and 1 for incumbent
- If entrant does not enter, gets payoff 1 while incumbent gets payoff 3

Entry Deterrence (cont)

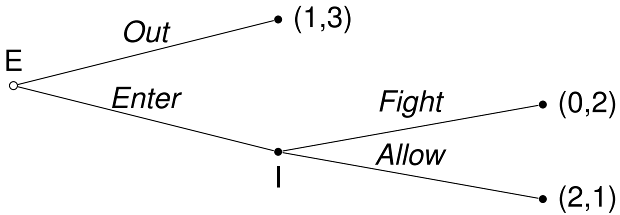
- How do we represent this game?



- What is SPNE of this game?
- Using backwards induction:
  - In last round, incumbent will not fight, since $1 > 0$
  - Knowing this, entrant will choose to enter, since $2 > 1$
  - SPNE is (Enter, Allow)
- Note that incumbent prefers entrant play Out, but cannot make *credible threat* that he will fight if entrant enters

Making the Threat Credible

- Suppose that incumbent monopolist has previously invested in technology that allows it to better fight off competition
  - If entrant enters and incumbent fights, payoffs now 0 for entrant and 2 for incumbent
- How do we represent this game?



- What is SPNE now?
  - Incumbent will choose fight if entrant chooses to enter
  - Knowing this, entrant decides to stay out (since $1 > 0$)
  - SPNE is (Out, Fight)

Penalty Kicks

- Consider a game between a penalty kicker and a goalie in soccer
- Kicker can kick either left or right
- Goalie simultaneously decides whether to defend left or right
- Suppose kicker's accuracy is as follows:
  - 50% if kick left and goalie defends left
  - 80% if kick left and goalie defends right
  - 90% if kick right and goalie defends left
  - 20% if kick right and goalie defends right
- Assume kicker payoff is probability that she scores
- This is a *zero-sum game*: Player's payoffs sum to zero in each outcome
- How do we represent this game?

	L	R
L	$(50, -50)$	$(80, -80)$
R	$(90, -90)$	$(20, -20)$

Penalty Kicks: Solution

- What is/are pure strategy Nash equilibrium/a?
  - None
- What is/are mixed strategy Nash equilibrium/a?
  - Assume kicker goes left with probability p and goalie goes left with probability q
  - Kickers indifference condition:

$$50q + 80(1-q) = 90q + 20(1-q)$$
$$q = 0.6$$

  - Goalie's indifference condition:

$$-50p - 90(1-p) = -80p - 20(1-p)$$
$$p = 0.7$$

  - Thus Nash equilibrium is $(p^*, q^*) = (0.7, 0.6)$