Econ 301: Microeconomic Analysis

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Social Welfare

Motivation

- Pareto efficiency says very little about the distribution of resources between people
- In general there are many Pareto efficient outcomes in an economy
- Pareto efficiency is a minimum standard for an allocation, but can we be more exacting?

Aggregation of Preferences

- Let x stand for the allocation of all goods in to all agents
- Each agent i has preference ordering over allocations given by $\succ_i$
  - In general, agents care about outcome for everyone, not just themselves
  - Assume that $\succ_i$ is complete, reflexive, and transitive for all agents i
- We seek a *social preference ordering* $\succ_s$ that aggregates the preferences of all agents
- We'll need a *mechanism* that takes all the individual orderings $\{\succ_i\}_{i=1}^{n}$ and returns a social ordering $\succ_s$

Example

- Three individuals: *A*, *B*, *C*
- Three outcomes: *x*, *y*, *z*
- Preferences:
  - *A*: $x \succ_A y \succ_A z$
  - *B*: $y \succ_B z \succ_B x$
  - *C*: $z \succ_C x \succ_C y$
- Can represent preferences like this:

A	B	C
x	y	z
y	z	x
z	x	y

- What mechanisms could we use to get social ordering from individual preferences?

Majority Vote

- One possible mechanism: pairwise majority vote
  - $x \succ_s y$ if a majority of subjects prefer *x* to *y*
- Recall:

A	B	C
x	y	z
y	z	x
z	x	y

- Will the resulting social preference relation be transitive?

Borda Count

- Another possible mechanism: let people report their rankings (1, 2, 3, ect) of each option
- Add up the rankings for each option
- Then $x \succ_s y$ if the aggregate ranking of *x* is lower than that of *y*
- Example:

A	B
x	y
y	z
z	x

- Suppose only *x* and *y* put on the table; which one wins?

- What if all three options put on the table?

Desirable Properties of Social Preference Orderings

1. If individual preferences are complete, reflexive, and transitive, then the social preference ordering should be as well.
   - Majority vote violates this
2. The social preference ordering between *x* and *y* should only depend on the how individuals rank *x* vs *y*.
   - Property sometimes called *independence of irrelevant alternatives (IIA)*
   - Borda count violates this
3. If $x \succ_i y$ for all individuals, then $x \succ_s y$
4. No dictatorship. A *dictatorship* is when there is some individual *i* such that $x \succ_s y$ if and only if $x \succ_i y$.

Arrow's Impossibility Theorem

> **Theorem (Arrow)**
>
> *There is no mechanism that satisfies properties 1 through 4.*

> **Theorem (Arrow, alternate version)**
>
> *The only mechanism that satisfies properties 1 through 3 is a dictatorship.*

- So what do we do?
  - If we drop property 2 (IIA), many voting mechanisms will satisfy remaining properties (such as Borda count)

Social Welfare Functions

- Instead of aggregating preference rankings, we can aggregate utility functions
- Define a *social welfare function* as

$$W(x) = W\left(u_1(x), u_2(x), \ldots, u_n(x)\right)$$

where n is number of individuals in economy
  - Only requirement is that W is increasing in each argument
  - Issue: utility functions are not unique, so we have to pick one representation
- Once we have picked social welfare function:

$$x \succ_s y \text{ if and only if } W(x) > W(y)$$

Example Social Welfare Functions

- Social welfare function can take many forms
- Possible social welfare functions
  - Classical:
  $$W(x) = \sum_{i=1}^{n} u_i(x)$$

  - Weighted sum:
  $$W(x) = \sum_{i=1}^{n} \alpha_i u_i(x)$$

  - Rawlsian:
  $$W(x) = \min\{u_1(x), u_2(x), \ldots, u_n(x)\}$$

Social Welfare Maximization

- Let there be k goods and n individuals in economy
- Assume there are amounts $X^1, \ldots, X^k$ of each good available via endowments
- Then the *social welfare maximization problem* or *social planner's problem* is

$$\max_x W(x) \text{ s.t. } \sum_{i=1}^{n} x_i^j = X^j \text{ for all } j \in \{1, 2, \ldots k\}$$

- Call solution a *maximal welfare allocation*

Visualizing Social Welfare Maximization

- For simplicity assume 2 agents
- Make axes u_1 and u_2
- *Utility possibilities set*: set of combinations of utility for agents 1 and 2 that are feasible given endowments
- Edge of this set is *utility possibilities frontier*
    - What is special about points on fronteir?
- *Isowelfare curves* trace out points in space that give same social welfare
    - What determines shape of isowelfare curves?

- Maximal welfare allocation is at point on utility possibility frontier that reaches highest isowelfare curve

Social Welfare Maximization

Welfare Maximization and Pareto Efficiency

- Will a maximal welfare allocation be Pareto efficient?

- Will a Pareto efficient allocation be a maximal welfare allocation (for some welfare function)?

Symmetric Allocations

- Consider an allocation that is *symmetric*, meaning that each agent gets equal amounts of each good
- Will this be Pareto efficient?
- Consider following example:
    - Three agents: A, B, C
    - Two goods: x_1 and x_2, one unit of each in economy
    - Agents A and B care only about having more of good 1
    - Agent C cares only about having more of good 2
- Symmetric allocation?

- Is this Pareto optimal?