

Motivation

- ▶ Much interesting economic behavior happens over many time periods
- ▶ Examples?

- ▶ We need a way to formally analyze these types of choices
 - ▶ For simplicity, assume small number of discrete time periods
 - ▶ Preferences do not change period-to-period, but relative impact of one period's consumption depends on how far in the future it is

Setup

- ▶ Two time periods: $t = 1, 2$
- ▶ A single good consumed in either period
 - ▶ Consumption c_1 and c_2 in periods 1 and 2, respectively
 - ▶ For now, assume price of consumption good is 1 in both periods
- ▶ Endowment $E = (m_1, m_2)$ in dollars
- ▶ Interest rate r
 - ▶ \$1 of endowment not spent in period 1 grows becomes $\$(1 + r)$ in period 2

Deriving the Budget Constraint

- ▶ Suppose you consume $c_1 < m_1$ in period 1
 - ▶ How much money stays in your bank account?
 - ▶ How much can you consume in period 2?
- ▶ Rearrange:
- ▶ Suppose you consume $c_1 > m_1$ in period 1
 - ▶ Instead of saving, borrow at interest rate r
 - ▶ How much money do you borrow?
 - ▶ How much do you have to pay back in period 2?
 - ▶ How much can you consume in period 2?
- ▶ Rearrange:
- ▶ Note same budget constraint whether save or borrow!

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The Intertemporal Budget Constraint

- ▶ The budget constraint (*future value* formulation):

$$(1 + r)c_1 + c_2 = (1 + r)m_1 + m_2$$

- ▶ The budget constraint another way (*present value* formulation):

$$c_1 + \frac{1}{1 + r}c_2 = m_1 + \frac{1}{1 + r}m_2$$

- ▶ These are the exact same formula
 - ▶ Just divide FV by $(1 + r)$ to get PV
- ▶ Why we have two ways of writing budget will be clear shortly

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Drawing the Budget Constraint

Future Value vs Present Value

- ▶ What is value of \$1 today in terms of consumption tomorrow?
 - ▶ Recall $(1 + r)c_1 + c_2 = (1 + r)m_1 + m_2$
 - ▶ One extra dollar today can buy $1 + r$ units of consumption tomorrow
 - ▶ Thus *future value* of \$1 is $\$(1 + r)$
- ▶ What is value of \$1 in the future in terms of consumption today?
 - ▶ Recall $c_1 + \frac{1}{(1+r)}c_2 = m_1 + \frac{1}{(1+r)}m_2$
 - ▶ One extra dollar in the future can buy $\frac{1}{(1+r)}$ units of consumption today
 - ▶ Thus *present value* of \$1 is $\$\frac{1}{(1+r)}$
- ▶ By convention, we typically work with present value (PV)
- ▶ Note budget constraint says $PV(\text{endowment}) = PV(\text{consumption})$
- ▶ Higher PV means the consumer can consume more in every period

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Higher PV Makes Consumer Better Off

Borrowing and Lending

- ▶ If $c_1 > m_1$, you are a *net borrower*
- ▶ If $c_1 < m_1$, you are a *net lender*

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Effect of Increasing Interest Rate

Effect of Decreasing Interest Rate

- ▶ What happens when interest rate r increases?
 - ▶ Budget line becomes
 - ▶ Budget line rotates around
 - ▶ What do borrowers do?
 - ▶ What do lenders do?

- ▶ What happens when interest rate r decreases?
 - ▶ Budget line becomes
 - ▶ Budget line rotates around
 - ▶ What do borrowers do?
 - ▶ What do lenders do?

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Intertemporal Utility

- ▶ Utility is $U(c_1, c_2) = u(c_1) + \frac{1}{1+\delta}u(c_2)$
- ▶ $\delta \geq 0$ is *discount factor*
- ▶ Non-zero δ means future consumption less valuable than today's
- ▶ The higher the δ , the less patient decision-maker is
- ▶ For example, suppose $u(c) = c$ and $\delta = 0.2$
 - ▶ Would DM rather have $(c_1, c_2) = (0, 12)$ or $(9, 0)$?
- ▶ But suppose $\delta = 0.5$ (more impatient)
 - ▶ Would DM rather have $(c_1, c_2) = (0, 12)$ or $(9, 0)$?

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Intertemporal Choices

- ▶ What is general maximization problem?
- ▶ What is solution?

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Intertemporal Choices

- ▶ Take the ratio of the FOC:

$$\frac{u'(c_1)}{u'(c_2)} = \frac{1+r}{1+\delta}$$

- ▶ Assume that $\frac{\partial u}{\partial c} > 0$ and $\frac{\partial^2 u}{\partial c^2} < 0$
- ▶ What happens if $\delta = r$?
- ▶ What happens if $\delta < r$?

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Allowing for Inflation

- ▶ What if different prices in each period: p_1 and p_2 ?
- ▶ Assume endowment m is in consumption units
- ▶ Suppose you consume c_1 in period 1
- ▶ How much money stays in your bank account?
- ▶ How much can you consume in period 2?
- ▶ Rearrange to find FV formulation of budget constraint:
- ▶ Divide through by $(1+r)$ to get PV formulation:

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Inflation and Real Interest Rates

- ▶ Let $\frac{p_2}{p_1} = 1 + \pi$, where π is the *inflation rate*
- ▶ Then PV formulation of budget becomes

- ▶ Let $\frac{1+r}{1+\pi} = 1 + \rho$, where ρ is the *real interest rate*:

- ▶ Looks just like non-inflation BC with r replaced by ρ
- ▶ Note: can also do BC with dollar-valued m 's instead of consumption-valued m 's

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Multiple Periods

- ▶ Suppose we have 3 periods (with same price)
- ▶ Budget constraint if can borrow or lend at rate r between two consecutive periods

$$c_1 + \frac{1}{1+r}c_2 + \frac{1}{(1+r)^2}c_3 = m_1 + \frac{1}{1+r}m_2 + \frac{1}{(1+r)^2}m_3$$

- ▶ What if interest rate changes each period?
 - ▶ Eg r_1 between periods 1 and 2, r_2 between periods 2 and 3

$$c_1 + \frac{1}{1+r_1}c_2 + \frac{1}{(1+r_1)(1+r_2)}c_3 = m_1 + \frac{1}{1+r_1}m_2 + \frac{1}{(1+r_1)(1+r_2)}m_3$$

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Net Present Value

- ▶ When choosing between two investment opportunities, choose one with higher present value
- ▶ May need to pay P_1 and P_2 to get income M_1 and M_2 in periods 1 and 2, respectively
- ▶ In this case, compare net income flows, ie *net present value*:

$$NPV = (M_1 - P_1) + \frac{1}{1+r}(M_2 - P_2)$$

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Example: Bonds

- ▶ Financial securities: instruments that pay out money over time
- ▶ Bonds are one type of security
 - ▶ Typically a way for governments or companies to borrow from consumers
 - ▶ Bond holder (consumer) pays for bond up front
 - ▶ Bond issuer pays *coupon* x every period until period T plus *face value* F in period T (*maturity date*)
- ▶ What is present value of a bond?

- ▶ What is price of a bond?

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