# Econ 301: Microeconomic Analysis

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# Cost Functions



## Cost Functions

- So far we have written the cost function $c(w_1, w_2, y)$
- From now on, we will use shorthand $c(y)$
- We will break the cost function into two parts:

$$c(y) = c_v(y) + F$$

  - $c_v(y)$ is the *variable cost*, also written $VC(y)$
  - $F$ is the *fixed cost*
- We can divide the cost function by the output to get the *average cost* function:

$$AC(y) = \frac{c(y)}{y} = \frac{c_v(y)}{y} + \frac{F}{y}$$

  - $\frac{c_v(y)}{y}$ is the *average variable cost*, $AVC(y)$
  - $\frac{F}{y}$ is the *average fixed cost*, $AFC(y)$



## Average Cost Graphically

- How does AFC depend on $y$?
- How does AVC depend on $y$?

- Adding these up, we see that AC is initially decreasing, then increasing, in $y$

## Marginal Cost

- Useful to define the *marginal cost* curve:

$$MC(y) = c'(y) = c_v'(y)$$

- MC is the slope of the cost function
- Measures how the cost of producing an additional unit of output changes as the total output level changes



## MC and AVC Have Same Vertical Intercepts

- MC curve and AVC curve approach the same height as $y$ approaches 0
- Formal proof:
  - The limit of the AVC curve as $y$ approaches 0 is

$$\lim_{y \to 0} \frac{c_v(y)}{y}$$

  - Note we can't just plug in 0, since numerator and denominator both go to zero
  - But using l'Hopital's rule, we can find the limit:

$$\lim_{y \to 0} AVC(y) = \lim_{y \to 0} \frac{c_v(y)}{y} = \frac{\lim_{y \to 0} c_v'(y)}{\lim_{y \to 0} 1} = \lim_{y \to 0} c_v'(y) = \lim_{y \to 0} MC(y)$$



## MC Intersects AVC at Minimum

- MC curve intersects the AVC curve where the AVC curve is minimized
- Consider range where AVC is decreasing
  - As output increases, must be adding a number smaller than the average to the total
  - Thus $MC(y) < AVC(y)$ in this range
- Consider range where AVC is increasing
  - As output increases, must be adding a number larger that the average to the total
  - Thus $MC(y) > AVC(y)$ in this range
- Thus $MC(y) = AVC(y)$ at the point where $AVC(y)$ is neither increasing nor decreasing, ie the minimum



## MC Intersects AC at Minimum

- The argument on the previous page applies to the average cost curve as well
- Thus we have $MC(y) = AC(y)$ at the point where $AC(y)$ is minimized
- Note because *AC* curve lies above *AVC* curve, minimum of AC curve is to the right of minimum of *AVC* curve

## Marginal Cost Graphically

- These three properties allow us to draw the MC curve given the AVC and AC curves:
  1. Marginal cost starts out at the same level as average variable cost
     - Note if AVC is initially decreasing, MC will initially decrease as well
  2. Marginal cost intersects AVC curve at AVC minimum
  3. Marginal cost intersects AC curve at AC minimum



## Marginal Cost Graphically



## Cost Curve Example

- Let $c(y) = y^2 + 1$
- What is variable cost?
- What is average variable cost?
- What is fixed cost?
- What is average fixed cost?
- What is average cost?
- What is marginal cost?



## Cost Curve Example, con't

- Where is minimum of *AVC* curve?
- Where is minimum of *AC* curve?

- Does the *MC* curve go through these points?

## Firm Supply



## Firm Constraints

- How much output the firm sells depends on two constraints
  1. Technology constraints
  2. Market constraints
- We have focused so far on technology
- It is now time to consider the role of the market
  - Many possible assumptions for what type of market we are in, eg monopoly, oligopoly
  - For now we will assume we are in a perfectly competitive market, ie firms are price takers



## Demand Curve Faced by the Firm

- We call the relationship between the price the firm sets and the amount it sells at that price the *demand curve faced by the firm*
- This is not the same as the market demand curve in general
  - Though that is true for monopoly
- What is the demand curve faced by the firm for perfect competition, assuming market price $p^*$?
- Thus under our assumption that firm is small (and hence cannot supply the whole market), they will price at the market price (if they sell at all)
- So we only have to worry about quantity decision, not pricing decision



## Demand Curve Faced by Firm in Competitive Market

## Supply Decision

- Assuming a cost function $c(y)$, the firm solves

$$\max_y py - c(y)$$

- The first order condition gives us

$$p = c'(y) = MC(y)$$

- That is, price (which equals marginal revenue) equals marginal cost
- The FOC gives us a relationship between market price and firm supply
- Thus $p = MC(y)$ defines the supply curve
  - with just two wrinkles ...



## Second-Order Condition

- Note that if the MC curve is downward sloping initially, we may have two points where $p = MC(y)$
- The lower of these two will not be maximizing, since at this point the firm could increase output to increase profits
- We can check that we are at a maximum, not a minimum, by the second-order condition, which is in this case

$$\frac{d^2\pi}{dy^2} = -c''(y) < 0$$



## Shutdown Condition

- So far we have assumed that it is optimal for firm to produce some positive amount
- Firm always has the option to shut down and produce nothing
- In this case they would pay only fixed cost $F$, so profit would be $-F$
- Should shut down if $py^* - c_v(y^*) - F < -F$, ie $py^* - c_v(y^*) < 0$
- Rearrange:

$$p < AVC(y^*)$$

- This is called the *shutdown condition*: firm should shut down if price is less than average variable cost at optimal quantity



## Supply Curve

- Shutdown condition kicks in when MC curve goes below AVC curve
- Thus the *supply curve* is given by the part of the MC curve that upward sloping and above the AVC curve
  - If $p < \min AVC$ then the supply is zero
- As usual we can define the *inverse supply curve* as the price as a function of quantity supplied

## Supply Curve Graphically



## Profit

- Suppose firm is facing price $p^*$ and producing optimal quantity $y^*$
- Profit is given by revenue minus costs:

$$\pi^* = p^* y^* - c(y^*) = p^* y^* - AC(y^*) y^* = [p^* - AC(y^*)] y^*$$

- This is area of rectangle of height $p^* - AC(y^*)$ and length $y^*$
- Note we can also write $\pi^* = p^* y^* - c_v(y^*) - F$



## Profit Graphically



## Producer Surplus

- We defined *producer surplus* as the area under the market price and above/left of the supply curve
- There is an equivalent definition: area of width $y^*$ and height equal to the difference between $MC(y^*)$ and $AVC(Y^*)$
- That is,

$$\begin{aligned} PS =& [MC(y^*) - AVC(y^*)] y^* \\ =& \left[p^* - \frac{c_v(y^*)}{y^*}\right] y^* \\ =& p^* y^* - c_v(y^*) \end{aligned}$$

- Notice that $PS = \pi + F$, which implies $\Delta PS = \Delta \pi$

## Different Ways to Calculate Producer Surplus



## Example

- Recall our example cost function $c(y) = y^2 + 1$
- What is the supply curve?
- What is the inverse supply curve?
- What is profit as a function of $p$?

- What is producer surplus as a function of $p$?



Appendix



## Marginal Cost and Variable Cost

- There is one more property of marginal cost that comes in handy
- The area under the MC curve up until output $y$ is equal to $VC(y)$
- Intuition: adding up the marginal cost to product output $y$ captures the variable cost up until that point, but not the fixed cost, ie the starting point
- The formal proof relies on the fundamental theorem of calculus:

$$\int_0^y MC(y')dy' = \int_0^y \frac{dc_v(y')}{dy'}dy' = c_v(y) - c_v(0) = c_v(y)$$