Econ 301: Microeconomic Analysis

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Labor Supply

Setting Up the Problem

- Suppose you have the utility function $U(C, H) = C^2 H$
  - C is amount of composite consumption good per day, price p
  - H is leisure time, in hours per day
- Can earn wage w per hour working out of possible hours $\bar{H}$ in the day
- Call L the hours you choose to work; remainder is leisure hours H
- Only source of income is from wages

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- What is opportunity cost of one hour of leisure in terms of the consumption good?
  - One extra hour of leisure means one less hour of work
  - That means w less income to spend
  - Could have bought $\frac{w}{p}$ units of consumption good with that income
  - Thus opportunity cost of hour of leisure is $\frac{w}{p}$

Leisure Demand

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- Setting up constrained optimization problem:

$$\mathcal{L} = C^2 H + \lambda(w\bar{H} - pC - wH)$$

- First order conditions:

$$2CH - \lambda p = 0 \qquad C^2 - \lambda w = 0 \qquad w\bar{H} - wH - pC = 0$$

- Take the ratio of these to find

$$\frac{2H}{C} = \frac{p}{w}$$

- Plug into the budget constraint:

$$p\left(2H\frac{w}{p}\right) + wH = w\bar{H}$$

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- Solve to find
  - Leisure demanded $H = \frac{\bar{H}}{3}$
  - Consumption demanded $C = \frac{2}{3}\frac{w}{p}\bar{H}$
- How does leisure demand depend on price of leisure (ie wage)?
  - Leisure demanded (or equivalently labor supplied) does not change with wages or price of consumption
  - Eg if $\bar{H} = 24$, you will work 16 hours a day regardless of the wage
  - This is the case for Cobb-Douglas utility function, though not in general

Income and Substitution Effects

Motivation

- We just derived that for Cobb-Douglas utility function, hours you work does not depend on your wage
- Suppose you have a job that pays \$10/hr and you decide to work 8 hours/day
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     - Have give up less of good 2 to get same amount of good 1
     - Causes more consumption of good 1 and less consumption of good 2 (holding purchasing power fixed)
  2. Income effect
     - Lower price of good 1 means more purchasing power overall
     - Causes more consumption of both goods (assuming goods are normal)

Graphical Decomposition

- Start with prices p_1, p_2, budget m, and demand $X = (x_1, x_2)$
- Want to find out what new demand will be at when price of good 1 changes to $p_1' < p_1$

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- The move from X to Y is the *substitution effect*
- The move from Y to Z is the *income effect*

Graphical Decomposition

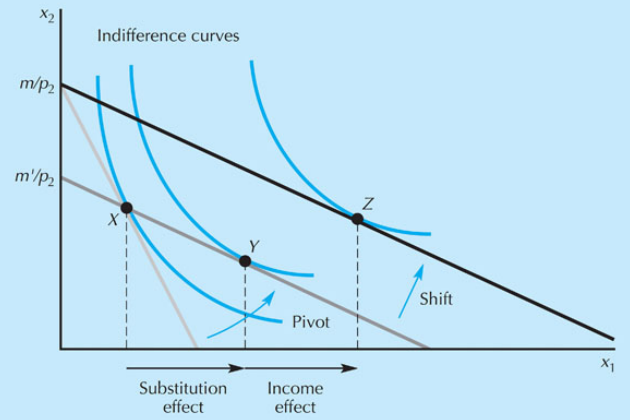

Figure 8.2

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  - Thus $p_1' < p_1$ leads to $\Delta x_1^s \geq 0$
- In general, we have $\frac{\Delta x_1^s}{\Delta p_1} \leq 0$
  - In calculus terms, that is $\frac{\partial x_1^s}{\partial p_1} \leq 0$

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  - Thus the sign of $\frac{\partial x_1^n}{\partial m} \approx \frac{\Delta x_1^n}{\Delta m}$ depends on whether good 1 is normal good

Total Change in Demand

- Note that the total change in demand is

$$\begin{aligned}\Delta x_1 &= x_1(p_1', m) - x_1(p_1, m)\\ &= x_1(p_1', m') - x_1(p_1, m) + x_1(p_1', m) - x_1(p_1', m')\\ &= \Delta x_1^s + \Delta x_1^n\end{aligned}$$

- This is one form of the *Slutsky identity* or *Slutsky equation*