Econ 301: Microeconomic Analysis

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Technology

Context

- We have so far assumed supply function is given
- To determine supply function, we need to talk about production technology
  - Current technology defines what inputs are required to produce a given input
  - Or conversely, how much output is feasible from given inputs
- Producer theory has many analogies to consumer theory
  - Inputs are like consumption bundles
  - Output is like utility, though we can't rescale outputs as we could with utility

Inputs and Outputs

- We call the good being produced the *output*
- *Inputs* or *factors* are the goods that go into the production process
  - Examples? land, materials, labor, machines
- One special kind of input is *capital*: inputs that are themselves outputs from another production process
  - Examples? machines, computers, vehicles
- Usually measure inputs and outputs as flows, eg widgets per month

Technology Constraints

- Technological constraints tell us which combination of inputs are outputs are possible
- How do we represent these constraints?
- One option: list all the pairs (x, y) that are in the *production set*
  - x is the input and y is the output
  - We can draw the set of such pairs
- Another option: write out the *production function*
  - Really care only about maximum possible output for given input
  - This is the upper boundary of the production set
  - This defines *production function* $y = f(x)$ where x is input and y is output

Production Set and Production Function

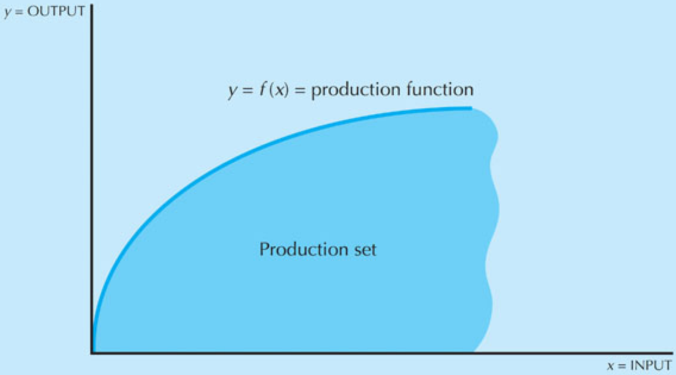

Figure 19.1 **A production set.** Here is a possible shape for a production set.

Isoquants

- More generally, production function may take more than one input, eg $y = f(x_1, x_2)$
  - Examples? Worker and shovels to dig ditches; steel and robots to make cars
- An *isoquant* is the set of all combinations of inputs that produce a given level of output
- That is, $\{x_1, x_2 | f(x_1, x_2) = \bar{y}\}$
- Analogous to indifference curves
  - Can't rescale isoquants like we did with indifference curves, however

Examples of Production Functions

- Fixed proportions
  - Production function: $f(x_1, x_2) = \min\{x_1, x_2\}$
  - Example? Digging a ditch with workers and shovels
  - Isoquants look like? Right angles
- Perfect substitutes
  - Production function: $f(x_1, x_2) = x_1 + x_2$
  - Example? Producing your homework with either red pen or blue pen
  - Isoquants look like? Straight lines
- Cobb-Douglas
  - Production function: $f(x_1, x_2) = Ax_1^a x_2^b$ where $A, a, b > 0$
  - A measures how many units of output we get for one unit each of inputs
  - Can't rescale exponents as in utility case

Properties of Production

Definition

A production technology is *monotonic* if $x_1 \geq z_1$ and $x_2 \geq z_2$ implies $f(x_1, x_2) \geq f(z_1, z_2)$. That is, output should increase if any inputs increase (and none decrease).

Definition

Suppose $f(x_1, x_2) = f(z_1, z_2) = \bar{y}$. Then a production technology satisfies *convexity* if $f(\alpha x_1 + (1 - \alpha)z_1, \alpha x_2 + (1 - \alpha)z_2) \geq \bar{y}$. That is, a combination of two sets of inputs increases output.

Marginal Product

- How much more output do we get from adding more of input *i*?
- We define the *marginal product* of input *i* as $MP_i = \frac{\partial f}{\partial x_i}$
- Note that all other inputs stay constant
- Analogous to marginal utility
- How does MP_i change as x_i increases?
  - We know $MP_i > 0$ for all x_i because of monotonicity
  - We typically assume that production increases at a decreasing rate
  - This is know as *diminishing marginal product*
  - Formall, we have $\frac{\partial^2 f}{\partial x_i^2} < 0$
  - Analogy: diminishing marginal utility

Technical Rate of Substitution

- Suppose we change the amount of input 1
- How much do we have to change the amount of input 2 to get the same level of output?
  - The slope of the isoquant curve tells us this
  - We call this the *technical rate of substitution*
  - $TRS(x_1, x_2) = -\frac{MP_1}{MP_2}$
  - Analogy: marginal rate of substitution
- We typically assume that we have *diminishing TRS*
  - As we increase amount of factor 1, need less of a change in factor 2 to stay on isoquant
  - Analogy: diminishing marginal rate of substitution

Deriving the TRS Formula

- Why is $TRS(x_1, x_2) = -\frac{MP_1}{MP_2}$ the formula for TRS?
  - Start with $y = f(x_1, x_2)$
  - Take differential: $dy = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2$
  - Set equal to zero and solve for total derivative:

$$\begin{aligned}
\frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 &= 0 \\
\frac{\partial f}{\partial x_2} dx_2 &= -\frac{\partial f}{\partial x_1} dx_1 \\
\frac{dx_2}{dx_1} &= -\frac{\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial x_2}} = -\frac{MP_1}{MP_2}
\end{aligned}$$

Returns to Scale

- What happens when we double inputs? Does output double as well?
- *Retuns to scale* indicate what happens when we increase all inputs by same amount
- Assuming $k > 1$:
  1. Constant returns to scale: $f(kx_1, kx_2) = kf(x_1, x_2)$
     - Example? Build an identical plant, get exactly twice as much output
  2. Increasing returns to scale: $f(kx_1, kx_2) > kf(x_1, x_2)$
     - Example? Double the diameter of a pipeline, get four times as much output
  3. Decreasing returns to scale: $f(kx_1, kx_2) < kf(x_1, x_2)$
     - Example? Double size of plant, get less than twice the output, perhaps because of communication complexity
- Returns to scale of a given production function can be different at different productions levels

Example: Cobb-Douglas

- Let $f(x_1, x_2) = Ax_1^a x_2^b$
- Does this demonstrate increasing or decreasing returns to scale?
  - Note that $f(kx_1, kx_2) = A(kx_1)^a(kx_2)^b = k^{a+b}Ax_1^a x_2^b = k^{a+b}f(x_1, x_2)$
  - Thus returns to scale are ...
    - increasing if $a + b > 1$
    - decreasing if $a + b < 1$
    - constant if $a + b = 1$

Long and Short Run

- In the *short run*, some inputs may be fixed at a certain level
- In the *long run*, all factors can be adjusted
- The length of these runs depends on context
- Example: farmer with fully-planted fields
  - Short run: land is fixed input
  - Long run: land can be bought and sold to re-optimize producer behavior

Example: Capital and Labor

- Suppose that the production function is $f(K, L) = K^{\frac{1}{4}} L^{\frac{1}{4}}$ where K is capital, L is labor
- What is the marginal product of capital?
  - $MP_K = \frac{\partial f}{\partial K} = \frac{1}{4} K^{-\frac{3}{4}} L^{\frac{1}{4}}$
- What is the marginal product of labor?
  - $MP_L = \frac{\partial f}{\partial K} = \frac{1}{4} K^{\frac{1}{4}} L^{-\frac{3}{4}}$
- What is the TRS of labor for capital?
  - $TRS = -\frac{MP_L}{MP_K} = -\frac{\frac{1}{4} K^{\frac{1}{4}} L^{-\frac{3}{4}}}{\frac{1}{4} K^{-\frac{3}{4}} L^{\frac{1}{4}}} = -\frac{K}{L}$
- What is the return to scale?
  - From earlier, since $a + b = \frac{1}{4} + \frac{1}{4} < 1$, decreasing returns to scale
- Which is the fixed factor in the short run? Typically capital