Econ 301: Microeconomic Analysis

Prof. Jeffrey Naecker

Wesleyan University

Exchange

Motivation

- So far in this class we have looked at one market at a time
  - Equilibrium in just one market (ignoring all others) is called *partial equilibrium*
- But in general what happens in one market will affect outcomes in other markets
  - So we move to study *general equilibrium*, which is equilibrium of all markets in the economy at the same time
- Simplifying assumptions:
  - Fully competitive markets
  - Just two markets and two consumers
  - Focus on *pure exchange* for now: trade with no production

Edgeworth Box: Setup

- We need to add a tool to our toolbox to tackle this problem
- Suppose two consumers, *A* and *B* and two goods, 1 and 2
- Consumers have initial *endowments* $\omega_A = (\omega_A^1, \omega_A^2)$ and $\omega_B = (\omega_B^1, \omega_B^2)$
- Consumers demand or *allocations* are $x_A = (x_A^1, x_A^2)$ and $x_B = (x_B^1, x_B^2)$
- An allocation (x_A, x_B) is *feasible* if $x_A^1 + x_B^1 = \omega_A^1 + \omega_B^1$ and $x_A^2 + x_B^2 = \omega_A^2 + \omega_B^2$

Drawing the Edgeworth Box

- Width of box: total amount of good 1 in economy: $\omega_A^1 + \omega_B^1$
- Height of box: total amount of good 2 in economy: $\omega_A^2 + \omega_B^2$
- Endowment $W = (\omega_A, \omega_B)$ is a point in the box
- Consumer A's allocation measured from lower left corner, which consumer B's endowment measured from upper right corner
- Consumer A's indifference curves open up and to the right, while consumer B's indifference curves open down and to the left

Edgeworth Box

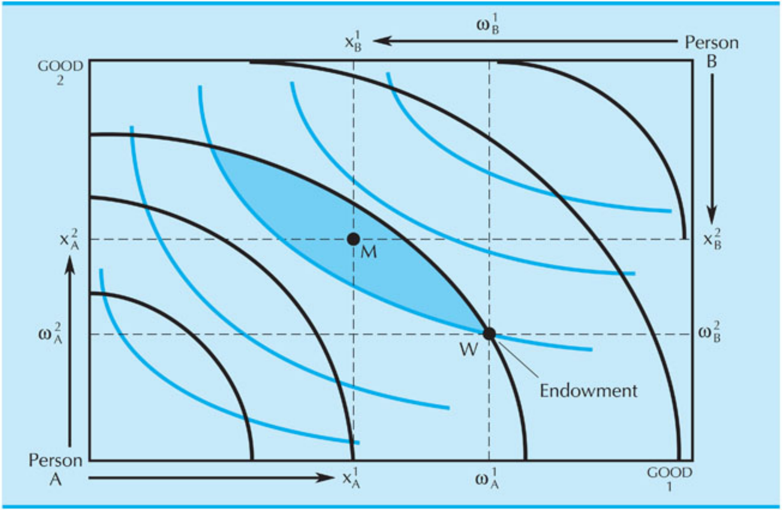

Figure 32.1

Trade in the Edgeworth Box

- Suppose that the consumers start at a point $W = (\omega_A, \omega_B)$ in the box
- Remember, an allocation is *Pareto efficient* if no one can be made better off without making someone worse off
- Is the endowment point is Pareto efficient?
  - Draw the indifference curves for both consumers that go through W
  - In general, there will be a lens-shaped area that is above A's indifference curve and below B's indifference curve
    - In this area, both consumers are better off than at endowment
  - Any trade that occurs should move consumers to a point in this region
  - Consumers are both better off anywhere in lens, so endowment is *not* Pareto efficient

Pareto Efficient Allocations in the Box

- Are there any allocations that are Pareto efficient?
  - Yes: an allocation is Pareto efficient if the two consumer's indifference curves are tangent at that point
- Is there more than one such point?
  - Yes, in general there will be a continuum of Pareto efficient points
  - We call this the *contract curve*
  - Note that the bottom left and upper right corners must be on the contract curve
- Thus any trade starting at the endowment must end up on the contract curve and inside the lens
  - We call this part of the contract curve the *core*

Contract Curve

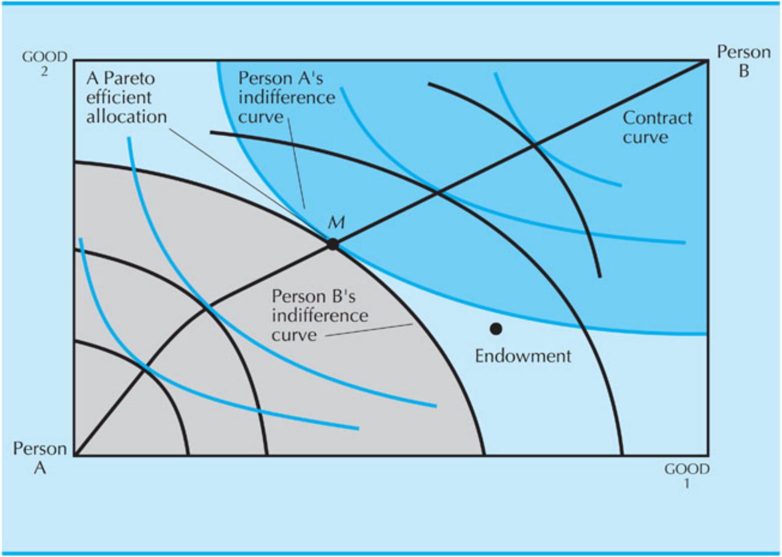

Figure 32.2

Adding Prices

- Imagine a neutral third party (often called the *auctioneer*) who sets prices $p = (p_1, p_2)$ for goods 1 and 2
- Based on preferences and budget, we can calculate each consumer's demand (sometimes called *gross demand*):

$$x_A = x_A(p, m_A) = (x_A^1(p, m_A), x_A^2(p, m_A))$$
$$x_B = x_B(p, m_B) = (x_B^1(p, m_B), x_B^2(p, m_B))$$

- We then define *excess* or *net demand* for each consumer:

$$e_A = (e_A^1, e_A^2) = (x_A^1 - \omega_A^1, x_A^2 - \omega_A^2)$$
$$e_B = (e_B^1, e_B^2) = (x_B^1 - \omega_B^1, x_B^2 - \omega_B^2)$$

The Budget Constraint

- Budget constraints for the two consumers are represented by the *same line* in the Edgeworth box

$$\begin{aligned} p_1 x_A^1 + p_2 x_A^2 &= p_1 \omega_A^1 + p_2 \omega_A^2 \\ \implies p_1(\omega_A^1 + \omega_B^1 - x_B^1) + p_2(\omega_A^2 + \omega_B^2 - x_B^2) &= p_1 \omega_A^1 + p_2 \omega_A^2 \\ \implies p_1 x_B^1 + p_2 x_B^2 &= p_1 \omega_B^1 + p_2 \omega_B^2 \end{aligned}$$

- Note that the endowment point is on the budget set implied by the prices: consumer could decide to just consume their endowment

Demand in the Edgeworth Box

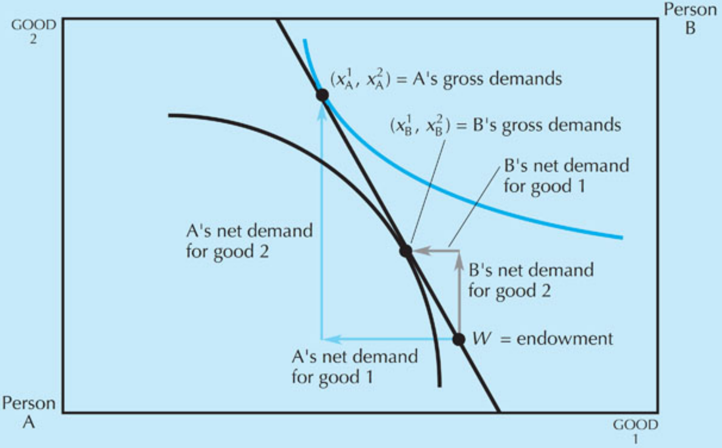

Figure 32.3

Competitive Equilibrium

- The economy is in *competitive equilibrium* (or *Walrasian equilibrium*) at prices $p^* = (p_1^*, p_2^*)$ and endowment $W = (\omega_A, \omega_B)$ when total demand equals total supply in each market
  - For 2-good economy, this means we have

$$x_A^1(p^*) + x_B^1(p^*) = \omega_A^1 + \omega_B^1$$
$$x_A^2(p^*) + x_B^2(p^*) = \omega_A^2 + \omega_B^2$$

  - This is called *market clearing* condition
- Note that since both consumers are optimizing, their indifference curves must be tangent to budget curve
  - And since they face the same prices, the indifference curves must also be tangent to each other
- Equilibrium is guaranteed to exist (as long as each consumer's demand is continuous, or each consumer is small relative to the market)

Aggregate Excess Demand

- We can define the *aggregate excess demand* for each good

$$z_1(p) = \underbrace{x_A^1(p) - \omega_A^1}_{e_A^1(p)} + \underbrace{x_B^1(p) - \omega_B^1}_{e_B^1(p)}$$

$$z_2(p) = \underbrace{x_A^2(p) - \omega_A^2}_{e_A^2(p)} + \underbrace{x_B^2(p) - \omega_B^2}_{e_B^2(p)}$$

- This gives us a new definition of competitive equilibrium:

$$z_1(p^*) = 0$$
$$z_2(p^*) = 0$$

- That is, aggregate excess demand of each good must be zero
- Each consumer wants to buy exactly as much as the other is selling (or vice versa)

Walras's Law

- Note consumer A's budget constraint can be written as

$$p_1 x_A^1 + p_2 x_A^2 = p_1 \omega_A^1 + p_2 \omega_A^2$$

- Rearranging, we get $p_1(x_A^1 - \omega_A^1) + p_2(x_A^2 - \omega_A^2) = 0$, or equivalently

$$p_1 e_A^1 + p_2 e_A^2 = 0$$

- Similarly, for B we will get $p_1 e_B^1 + p_2 e_B^2 = 0$
- Adding A and B's conditions together give

$$p_1(e_A^1 + e_B^1) + p_2(e_A^2 + e_B^2) = 0$$

  or

$$p_1 z_1 + p_2 z_2 = 0$$

- This last expression is known as *Walras's Law*

Using Walras's Law

- Note that if $z_1 = 0$ then Walras's Law gives $p_2 z_2 = 0$
- As long as $p_2 > 0$ this implies $z_2 = 0$ as well
- Clearly by same logic if $z_2 = 0$ we can immediately conclude that $z_1 = 0$ (as long as $p_1 > 0$)
- This means that for equilibrium it is sufficient to check just one of the two excess demand conditions
- In general, if we have k goods and $k - 1$ of them are in equilibrium, the kth market will be in equilibrium as well
- Note that we are therefore free to set price of one good equal to 1 (the *numeraire good*)

Equilibrium in the Edgeworth Box

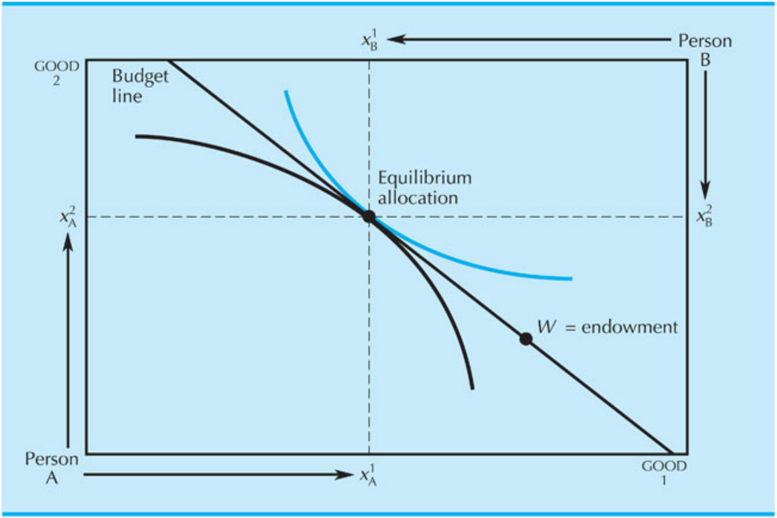

Figure 32.4

Equilibrium Example

- Suppose both consumer have Cobb-Douglas preferences:

$$u_A(x_A^1, x_A^2) = (x_A^1)^a (x_A^2)^{1-a}$$
$$u_B(x_B^1, x_B^2) = (x_B^1)^b (x_B^2)^{1-b}$$

- What is (gross) demand for the two consumers?

$$x_A^1(p, m_A) = a\frac{m_A}{p_1} \qquad x_B^1(p, m_B) = b\frac{m_B}{p_1}$$
$$x_A^2(p, m_A) = (1-a)\frac{m_A}{p_2} \qquad x_B^2(p, m_B) = (1-b)\frac{m_B}{p_2}$$

where $m_i = p_1\omega_i^1 + p_2\omega_i^2$ for $i = A, B$

Equilibrium Example (continued)

- What are equilibrium prices p_1^* and p_2^*?
- Must set $z_1 = 0$
- Thus equilibrium requires we have

$$\begin{aligned} 0 = z_1 &= x_A^1 - \omega_A^1 + x_B^1 - \omega_B^1 \\ &= \frac{a}{p_1}(p_1\omega_A^1 + p_2\omega_A^2) - \omega_A^1 + \frac{b}{p_1}(p_1\omega_B^1 + p_2\omega_B^2) - \omega_B^1 \end{aligned}$$

- Setting $p_2^* = 1$ (numeraire) and solving for p_1 we get

$$p_1^* = \frac{a\omega_A^2 + b\omega_B^2}{(1-a)\omega_A^1 + (1-b)\omega_B^1}$$

Contract Curve

- What is formula for contract curve in this example?
  - Note that contract curve is where indifference curves are tangent
  - Slope of indifference curve is MRS
  - $MRS_A = \frac{a}{1-a}\frac{x_A^2}{x_A^1}$ and $MRS_B = \frac{b}{1-b}\frac{x_B^2}{x_B^1} = \frac{b}{1-b}\frac{\omega_A^2+\omega_B^2-x_A^2}{\omega_A^1+\omega_B^1-x_A^1}$
  - Setting these equal, will get equation for contral curve

First Welfare Theorem

- Will a competitive equilibrium be Pareto efficient?
- Suppose equilibrium (x_A, x_B, p_1, p_2) was <u>not</u> Pareto efficient
- Then there must exist allocation (y_A, y_B) that is both feasible and desirable for both consumers:

$$y_A^1 + y_B^1 = \omega_A^1 + \omega_B^1$$
$$y_A^2 + y_B^2 = \omega_A^2 + \omega_B^2$$

- For (x_A, x_B) to be optimal it must be that (y_A, y_B) was not affordable:

$$p_1 y_A^1 + p_2 y_A^2 > p_1 \omega_A^1 + p_2 \omega_A^2$$
$$p_1 y_B^1 + p_2 y_B^2 > p_1 \omega_B^1 + p_2 \omega_B^2$$

First Welfare Theorem (continued)

- Adding these last two equations together we get

$$p_1(y_A^1 + y_B^1) + p_2(y_A^2 + y_B^2) > p_1(\omega_A^1 + \omega_B^1) + p_2(\omega_A^2 + \omega_B^2)$$

- Plugging in the feasibility condition we get

$$p_1(\omega_A^1 + \omega_B^1) + p_2(\omega_A^2 + \omega_B^2) > p_1(\omega_A^1 + \omega_B^1) + p_2(\omega_A^2 + \omega_B^2)$$

- Clearly a contradiction
- Thus it must be that any competitive equilibrium is Pareto efficient
  - This is known as the *First Welfare Theorem*
- Huge implication: Market process will automatically find an efficient outcome (though not necessarily a fair one)

Second Welfare Theorem

- OK, so the First Welfare Theorem says that a competitive equilibrium is Pareto efficient
- Is the converse true? That is, are all Pareto efficient allocations possible equilibria?
- Yes, any Pareto efficient allocation can be a competitive equilibrium for some prices p and endowments W
  - This is the *Second Welfare Theorem*
  - Guaranteed as long as preferences are convex
  - Intuition: for Pareto efficiency, indifference curves are tangent, so we can find prices and endowment to run a budget curve right through the tangency point
- Huge implication: To get a desired efficient market outcome, just have to choose starting endowment and let market forces do their work