Econ 301: Microeconomic Analysis

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Intertemporal Choice

Motivation

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- Examples?
  - Retirement savings
  - Long-term investments by businesses
  - College savings (and returns to college education)
  - Stock market choices
- We need a way to formally analyze these types of choices
  - For simplicity, assume small number of discrete time periods
  - Preferences do not change period-to-period, but relative impact of one period's consumption depends on how far in the future it is

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- Endowment $E = (m_1, m_2)$ in dollars
- Interest rate r
  - \$1 of endowment not spent in period 1 grows becomes $\$(1 + r)$ in period 2

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- Note same budget constraint whether save or borrow!

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  - Just divide FV by $(1 + r)$ to get PV
- Why we have two ways of writing budget will be clear shortly

Drawing the Budget Constraint

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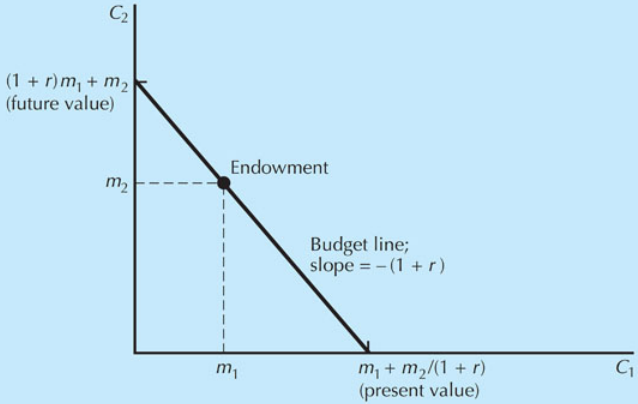

Figure 10.2

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- By convention, we typically work with present value (PV)
- Note budget constraint says PV(endowment) = PV(consumption)
- Higher PV means the consumer can consume more in every period

Higher PV Makes Consumer Better Off

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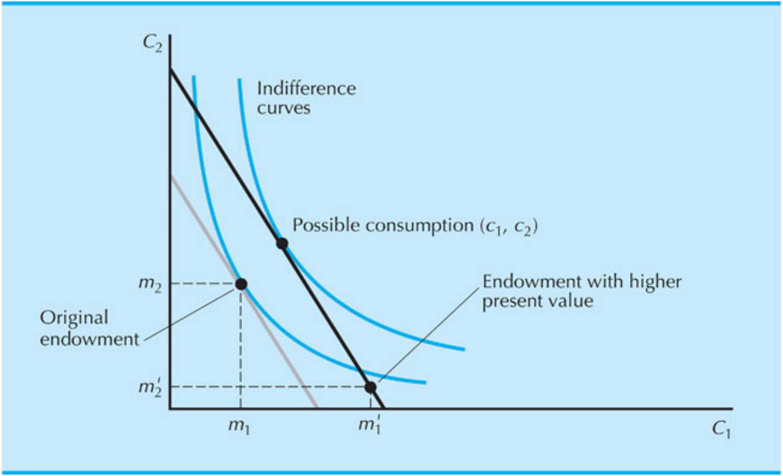

Figure 10.6

Borrowing and Lending

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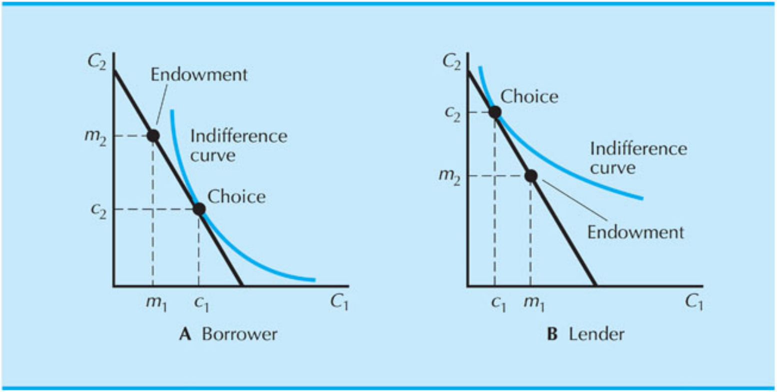

Effect of Increasing Interest Rate

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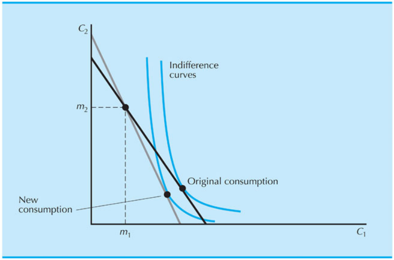

Figure 10.5

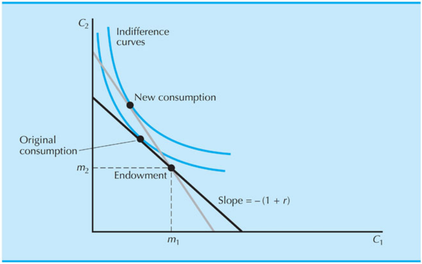

Figure 10.4

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- Utility is $U(c_1, c_2) = u(c_1) + \frac{1}{1+\delta} u(c_2)$
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  - That is, DM gives up 3 units to get consumption earlier

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- Set up Lagrangian:

$$\mathcal{L} = u(c_1) + \frac{1}{1+\delta} u(c_2) + \lambda \left(m_1 + \frac{1}{1+r} m_2 - c_1 - \frac{1}{1+r} c_2 \right)$$

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- FOC:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_1} = 0 \rightarrow & \quad u'(c_1) = \lambda \\ \frac{\partial \mathcal{L}}{\partial c_2} = 0 \rightarrow & \quad u'(c_2) = \lambda \tfrac{1+\delta}{1+r} \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \rightarrow & \quad \text{budget constraint} \end{aligned}$$

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$$\frac{u'(c_1)}{u'(c_2)} = \frac{1+r}{1+\delta}$$

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$$u'(c_1) > u'(c_2) \rightarrow c_1^* < c_2^*$$

Allowing for Inflation

- What if different prices in each period: p_1 and p_2?
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- Note: can also do BC with dollar-valued m's instead of consumption-valued m's

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Net Present Value

- When choosing between two investment opportunities, choose one with higher present value
- May need to pay P_1 and P_2 to get income M_1 and M_2 in periods 1 and 2, respectively
- In this case, compare net income flows, ie *net present value*:

$$NPV = (M_1 - P_1) + \frac{1}{1+r}(M_2 - P_2)$$

Example: Bonds

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- Bonds are one type of security
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  - Same as present value, so that NPV is zero