Econ 301: Microeconomic Analysis

Prof. Jeffrey Naecker

Wesleyan University

Oliogopoly

Motivation

- Today we will apply game theory to markets in particular
- Ideal tool to study the case where we have multiple firms interacting (so not monopoly) but firms are big enough to influence market price (so not pure competition)
  - This is *oligopoly*
- Two simplifying assumptions
  - Just two firms (*duopoly*)
  - Firms are producing identical products (no *product differentiation*)

Overview

- Two possible timings of firm choices
  - Sequential: use backwards induction
  - Simultaneous: use Nash equilibrium
- Two possible choice variables for firms:
  - Price
  - Quantity
- Thus there are four possible models:
  1. Sequential quantity competition (Stackleberg)
  2. Sequential price competition (won't cover this)
  3. Simultaneous quantity competition (Cournot)
  4. Simultaneous price competition (Bertrand)

Sequential Quantity Competition: Setup

- Firm 1 chooses quantity y_1 first (the *quantity leader* or *first mover*)
- Then firm 2 (the *follower* or *second mover*) chooses its quantity y_2
- Total quantity $Y = y_1 + y_2$
- Inverse demand $p(Y) = p(y_1 + y_2)$
- Cost functions $c_1(y_1)$ and $c_2(y_2)$
- Also known as *Stackleberg model*

Sequential Quantity Competition: Solution

- Which solution concept do we use?
  - Backwards induction, since this is sequential game
- Follower (firm 2) solves

$$\max_{y_2} p(y_1 + y_2)y_2 - c_2(y_2)$$

  - Gives us firm 2's *reaction function*: $y_2 = f_2(y_1)$
- Now consider firm 1's (leader's) problem:

$$\max_{y_1} p\left(y_1 + f_2(y_1)\right) y_1 - c_1(y_1)$$

  - Note that firm 2's reaction function is in firm 1's problem
  - Solving this problem gives leader's quantity
  - Plug in to find follower's quantity

Example: Linear Demand

- Suppose inverse demand is given by $p(Y) = a - bY = a - b(y_1 + y_2)$
- Assume both firms have zero marginal cost
- What are Stackleberg equilibrium quantities?
- Firm 2's problem:
  - Solve $\max_{y_2} [a - b(y_1 + y_2)]y_2$
  - Reaction function: $y_2 = \frac{a - by_1}{2b}$
- Firm 1's problem:
  - Solve $\max_{y_1} [a - b(y_1 + \frac{a - by_1}{2b})]y_1$
  - FOC: $\left[a - by_1 - b\left(\frac{a - by_1}{2b}\right)\right] + \left[-b + \frac{-b}{2}\right] y_1 = 0$
  - Solution: $y_1^* = \frac{a}{2b}$
- Then $y_2^* = \frac{a - b\frac{a}{2b}}{2b} = \frac{a}{4b}$
- Total quantity: $Y^* = y_1^* + y_2^* = \frac{3}{4}\frac{a}{b}$
- Note that there is *first mover advantage*

Simultaneous Quantity Competition

- Suppose firms both set quantity at same time
- What solution concept should we use now? Nash equilibrium
- Firm 1's problem:

$$\max_{y_1} p(y_1 + y_2)y_1 - c_1(y_1)$$

  - Can be solved to give optimal y_1 as function of y_2: $y_1 = f_1(y_2)$
  - This is reaction function, or best response function
- Similarly, can get firm 2's best response function: $y_2 = f_2(y_1)$

Definition

The Nash equilibrium of the Cournot model (known as Cournot equilibrium) is a quantity pair (y_1^*, y_2^*) such that

$$y_1^* = f_1(y_2^*)$$
$$y_2^* = f_2(y_1^*)$$

Cournot Equilibrium: Example

- Firms face linear inverse demand $p = a - bY$, have zero marginal cost
- What are Cournot equilibrium quantities?
- Firm 1 solves

$$\max_{y_1} [a - b(y_1 + y_2)] \, y_1$$

- We saw the solution to this already in sequential quantity competition:

$$y_1 = \frac{a - by_2}{2b}$$

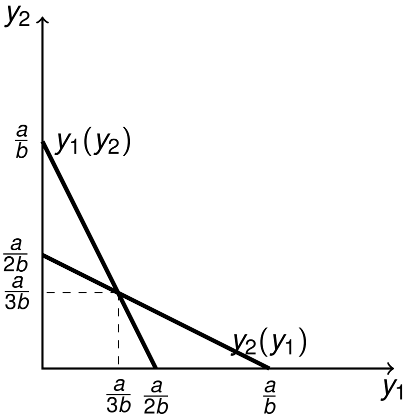
- Firm 2's problem is identical, so we know immediately that

$$y_2 = \frac{a - by_1}{2b}$$

- In equilibrium, have $y_1^* = y_2^*$ (only works when firms are identical)
- Thus we can solve to find $y_1^* = y_2^* = \frac{a}{3b}$

Cournot Equilibrium Graphically

- Note that Cournot equilibrium occurs where reaction functions/best response function cross



Simultaneous Price Setting

- Suppose instead firms simultaneously announce prices p_1 and p_2
- Both firms have constant marginal cost c
- Both firms have capacity to serve entire market
- Firm that announces lower price gets all of market share; if tie, they split market share
- What is the Nash equilibrium?
- Can firms announce prices that are above c in equilibrium?
  - No; if so, one or both firms would have incentive to undercut its competitor
- Can firms announce prices that are below c in equilibrium?
  - No; if so, one or both firms would have incentive to raise prices
- Only possibility: both firms announce $p_1 = p_2 = c$
- Note that this is the pure competitive equilibrium price!