Econ 301: Microeconomic Analysis

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Profit Maximization

Motivation

- Technology tells us which combinations of input and outputs are *possible*
- So how do firms pick which set of inputs to actually use?
- They maximize profits
- Assumption throughout this section: firms are price-takers both for selling their output and buying their inputs

Profits

- Setup:
  - Firm output y at price p
  - Firm inputs x_1 and x_2 at prices w_1 and w_2
- Revenue: py
- Cost: $w_1 x_1 + w_2 x_2$
- Profit = revenue - cost:

$$\pi = py - w_1 x_1 - w_2 x_2$$

- Profits and costs typically measured in flows, e.g. wage per month

Opportunity Costs

- Need to be careful to fully capture all inputs
  - If you are self-employed, labor is still an input
  - Price is implicit: what you could get if you worked for someone else
  - Same with rental rate of land, buildings, capital
- In short, opportunity costs are still costs

Profit Maximization

- The firm's profit maximization problem is

$$\max_{x_1,x_2} pf(x_1, x_2) - w_1 x_1 - w_2 x_2$$

- Taking first order conditions, we get

$$MP_1(x_1^*, x_2^*) = \frac{w_1}{p}$$

$$MP_2(x_1^*, x_2^*) = \frac{w_2}{p}$$

- Solving these for x_1^* and x_2^* gives *factor demand curves*

$$x_1^*(w_1, w_2, p)$$
$$x_2^*(w_1, w_2, p)$$

Marginal Product Equals Marginal Cost

- Note that FOC for input i is $MP_i = \frac{w_i}{p}$
- Note that $\frac{w_i}{p}$ is the marginal cost of input i (in terms of output good)
- Thus we can re-state FOC as $MP_i = MC_i$,
- That is, marginal product must equal marginal cost (in each output dimension) for firm to be maximizing
- You may have heard marginal *revenue* equals marginal cost
  - Note that marginal revenue $MR_i = pMP_i$, so our FOC can also be stated as $MR_i = w_i$
  - This is the same statement as above but in dollars instead of product units

Profit Maximization Intuition

- Can firm be profit-maximizing if $MP_1 > \frac{w_1}{p}$?

- Can firm be profit-maximizing if $MP_1 < \frac{w_1}{p}$?

Profit Maximization Graphically

- Consider case of just one input, x
- Consider fixed profit $\bar{\pi} = py - wx$
  - We can draw this as $y = \frac{\bar{\pi}}{p} + \frac{w}{p}x$
  - This is *isoprofit curve*
- Recall production function $f(x)$
- Profit maximization is equivalent to finding point on production function that hits highest isoprofit curve
  - Slope of production function:
  - Slope of isoprofit curve:
  - Tangency condition:

Profit Maximization Graphically

Example

- Given production function $f(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$, what are factor demands? (output price p, input prices w_1, w_2)

Example, cont

- What is optimal amount of production?

- What is profit?

Short Run vs Long Run

- So far, we have assumed that we are in the long run, since all input are variable
- In short run, some inputs may be fixed
- For example, fix $x_2 = \bar{x}_2$
- The firm's profit maximization problem is

$$\max_{x_1} pf(x_1, \bar{x}_2) - w_1 x_1 - w_2 \bar{x}_2$$

- Only one first-order condition:

$$MP_1(x_1^*, \bar{x}_2) = \frac{w_1}{p}$$

- What happens to factor demands if w_2 goes up?

Returns to Scale

- Firm finds optimal factor demands x_1^*, x_2^*, giving production y^*
- Making profit $\pi^* = py^* - w_1 x_1^* - w_2 x_2^*$
- Assume firm has constant returns to scale
- What happens to profit if firm doubles input levels?

- So what are only two possibilities?

Cost Minimization

Motivation

- Alternative way to figure out firm's optimal inputs and outputs: do in two steps
    1. Minimize cost given a level of output
    2. Choose optimal output
- For now, we focus on step one: cost minimization
    - Turns out this step will help us derive supply function

Cost Minimization Problem

▸ Cost minimization problem is given by

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \text{ s.t. } f(x_1, x_2) = y$$

▸ In words: choose inputs to find cheapest way to make output equal y

▸ Note that for this part, we think of y as a constant

Cost Minimization Solution

▸ Solve by setting up the Lagrangian:

$$\mathcal{L} = w_1 x_1 + w_2 x_2 + \lambda(y - f(x_1, x_2))$$

▸ Then take FOC:

$$w_1 = \lambda \frac{\partial f}{\partial x_1}$$

$$w_2 = \lambda \frac{\partial f}{\partial x_2}$$

$$y = f(x_1, x_2)$$

▸ Note that we can take ratio of first two FOC:

$$\frac{w_1}{w_2} = \frac{\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial x_2}} = \frac{MP_1}{MP_2} = -TRS$$

Cost Function

▸ Solving for the optimal inputs, we get the *conditional factor demand*:

$$x_1(w_1, w_2, y)$$
$$x_2(w_1, w_2, y)$$

▸ Called this because they are conditional on the output level y

▸ The formula for the minimized cost is called the *cost function*:

$$c(w_1, w_2, y) = w_1 x_1(w_1, w_2, y) + w_2 x_2(w_1, w_2, y)$$

Cost Minimization Graphically

▸ Fix cost at some level $c = \bar{c}$
  ▸ We can rearrange cost function $c = w_1 x_1 + w_2 x_2$ to find

$$x_2 = \frac{\bar{c}}{w_2} - \frac{w_1}{w_2} x_1$$

  ▸ This is an *isocost curve*
▸ Recall isoquant curve for fixed $y = \bar{y}$
▸ Cost minimization is equivalent to finding point on isoquant that touches lowest isocost curve
  ▸ Slope of isoquant:
  ▸ Slope of isocost:
  ▸ Tangency condition:

Cost Minimization Graphically

Examples of Cost Functions

- ▶ Perfect complements: $f(x_1, x_2) = \min\{x_1, x_2\}$
  - ▶ Conditional factor demand:

  - ▶ Cost function:

- ▶ Perfect substitutes: $f(x_1, x_2) = x_1 + x_2$
  - ▶ Conditional factor demand:

  - ▶ Cost function: