Econ 301: Microeconomic Analysis

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Preferences

Consumption Bundles

- Goods: things you can own or consume
- Consumption bundles: combinations of goods
  - Can in theory be very large, ie many goods
  - Simplify: two goods called 1 and 2 in amounts x_1 and x_2, respectively
  - Consumption bundle notation: $X = (x_1, x_2)$ or $Y = (y_1, y_2)$, eg
    - Capital letters for bundles
    - Subscript numbers indicate which good

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    - Capital letters for bundles
    - Subscript numbers indicate which good
- Example consumption bundles:
  - (1 car, 5 bananas)
  - (3 left shoes , 2 right shoes)
  - (2 hot dogs, 1.5 slices of pizza)

Preferences

- The simplest comparison we can make is between two bundles
- So we need a *binary relation*
- We have a few options:
  - $X \succ$Y means X is *strictly preferred* to Y
  - $X \sim$Y means X is *equivalent* to Y (usually say agent is *indifferent*)
  - $X \succeq$Y means X is *weakly preferred* or X *at least as good as* Y
- By convention, we usually use $\succeq$ as our fundamental relation
  - Note we can derive the others from it
  - If $X \succeq Y$ and $Y \succeq X$, then $X \sim Y$
  - If $X \succeq Y$ and *not* $Y \succeq X$, then $X \succ Y$

Properties of Rational Preferences Relations

- We can compare any two bundles:

Definition

A preference relation is *complete* if for any bundles X and Y, we have $X \succeq Y$, $Y \succeq X$, or both

- Note this means compare any bundle to itself (mostly for technical reasons)

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Definition

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- Note this means compare any bundle to itself (mostly for technical reasons)
- We also need the ordering to be logically consistent:

Definition

A preference relation is *transitive* if for any <u>distinct</u> bundles X, Y, and Z, if $X \succeq Y$ and $Y \succeq Z$, then we have $X \succeq Z$

Why These Axioms?

- Without these, there might be no “best option” from a list of consumption bundles
  - Incomplete: Might not be able to pick preferred bundle out of set $\{X, Y\}$
  - Intransitive: Even if we can make all pairwise comparisons, might not be able to choose from set of 3
- Note: these are *axioms* (guaranteed to hold) of the theory but *assumptions* (may or may not hold) about behavior

Example

- Consider a choice set of $\{A, B, C\}$ for apples, bananas, and carrots
- Suppose we have the following preference relation between the three bundles

$$\begin{array}{ccc} A \succeq A & A \succeq B & A \succeq C \\ B \succeq B & B \succeq C & C \succeq C \end{array}$$

- We can represent this relation with a matrix like this:

$$\begin{array}{c|ccc} & A & B & C \\ A & \cdot & \cdot & \cdot \\ B & & \cdot & \cdot \\ C & & & \cdot \end{array}$$

where the $\cdot$ in the first row, second column means that $A \succeq B$, etc

Example, con't

$$\begin{array}{cccc} & A & B & C \\ A & \cdot & \cdot & \cdot \\ B & & \cdot & \cdot \\ C & & & \cdot \end{array}$$

- Is this relation complete?

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$$\begin{array}{cccc} & A & B & C \\ A & \cdot & \cdot & \cdot \\ B & & \cdot & \cdot \\ C & & & \cdot \end{array}$$

- Is this relation complete? Yes, there is at least one dot relating each pair of letters (including self-pairs)

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	A	B	C
A	$\cdot$	$\cdot$	$\cdot$
B		$\cdot$	$\cdot$
C			$\cdot$

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- Is this relation transitive?

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$$\begin{array}{cccc} & A & B & C \\ A & \cdot & \cdot & \cdot \\ B & & \cdot & \cdot \\ C & & & \cdot \end{array}$$

- Is this relation complete? Yes, there is at least one dot relating each pair of letters (including self-pairs)
- Is this relation transitive? Yes, have to check all pairs of non-diagonal points
  - $A \succeq B$ and $A \succeq C$: antecedent does not bind
  - $A \succeq B$ and $B \succeq C$, so we need $A \succeq C$, which we have ✓
  - $A \succeq C$ and $B \succeq C$: antecedent does not bind

Indifference Curves

Indifference Curves

Suppose we plot all possible bundles on the cartesian grid

Definition

The *weakly preferred set* for some bundle $(\hat{x}_1, \hat{x}_2)$ is the set of all (x_1, x_2) such that $(x_1, x_2) \succeq (\hat{x}_1, \hat{x}_2)$

Definition

The *indifference curve* for some bundle $(\hat{x}_1, \hat{x}_2)$ is the set of all (x_1, x_2) such that $(x_1, x_2) \sim (\hat{x}_1, \hat{x}_2)$

- This is the edge of the weakly preferred set

Weakly Preferred Sets and Indifference Curves

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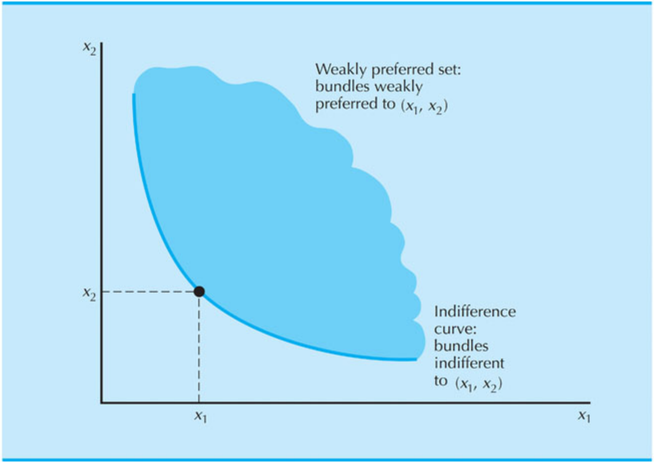

Figure 3.1

Crossing Indifference Curves

- Can indifference curves cross?

Crossing Indifference Curves

- Can indifference curves cross? No. Suppose they did:

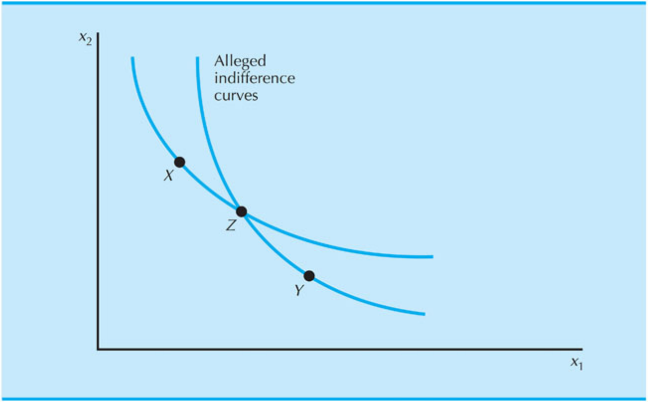

Figure
3.2

- Apparently $X \sim Z \sim Y$ but also $Z \succ X$ and $Z \succ Y$!

Monotonicity

Definition

A preference relation satisfies *monotonicity* if $y_1 \geq x_1$ and $y_2 \geq x_2$ imply $(y_1, y_2) \succeq (x_1, x_2)$

- Intuition: more is better
- What does monotonicity imply about indifference curves?

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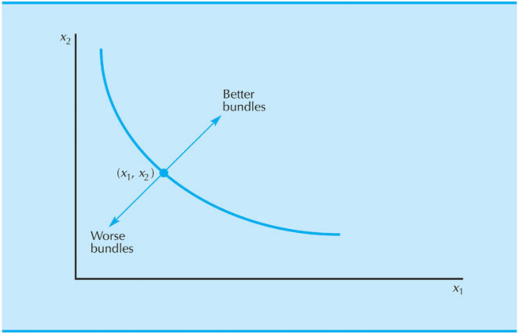

Convexity

Definition

A preference relation satisfies *convexity* if for any bundles $(x_1, x_2) \sim (y_1, y_2)$ and any $\alpha \in [0, 1]$ we have $(\alpha y_1 + (1-\alpha)x_1, \alpha y_2 + (1-\alpha)x_2) \succeq (x_1, x_2)$

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- What does convexity imply about weakly preferred sets?
  - Weakly preferred sets are convex

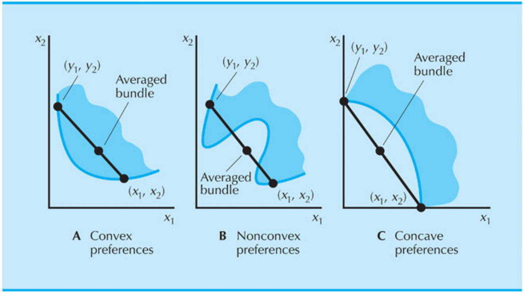

Figure 3.10

Utility

Utility

Definition

An *utility function* is a function u that assigns a real number to every bundle such that $u(x_1, x_2) \geq u(y_1, y_2)$ if and only if $(x_1, x_2) \succeq (y_1, y_2)$

- Derived from preferences, which are considered the fundamental object
- Preferences only tell us the order or ranking of bundles, so we usually talk about *ordinal* utility

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- Does a utility function exist for every possible preference ranking?
  - No: can't have utility function for intransitive preferences
- Assuming complete and transitive preferences, a utility function (or really functions) will exist
- A simple construction of a utility function: label all indifference curves with increasing numbers
  - Indifference curves are *level sets* utility functions
  - That is, an indifference curve is the set of points (x_1, x_2) such that $u(x_1, x_2) = k$ for some k

A Family of Utility Functions

- All of these utility functions U_1 through U_4 represent the same preferences:

bundle	U_1	U_2	U_3	U_4
A	3	4	17.85	-1
B	2	3	11.20	- 2
C	1	2	0.01	-3

- All we are doing is relabeling indifference curves
- From one preference ordering we can arrive at many utility functions
- This is why we only concern ourselves with *ordinal* information in utility functions

Monotone Transformations

Definition

A function f is *monotonic* if $f(u_2) - f(u_1) \geq 0$ for any u_1 and u_2 such that $u_2 \geq u_1$. That is, it is *weakly increasing*.

Theorem

If $f(u)$ is a monotonic function, and $u(x_1, x_2)$ is a utility function representing some preference ordering, then $f(u(x_1, x_2))$ is also a utility function representing that same preference ordering

What does this mean practically?

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What does this mean practically? Can apply any monotonic function (eg taking logs, adding a constant) to utility function if it makes math easier

Marginal Rates of Substitution

- The marginal rate of substitution (MRS) from good 1 to good 2 is the exchange rate at which the consumer would be just indifferent about trading
- In terms of indifference curves: MRS is the slope of the indifference curve (may be changing)
- In terms of the utility function:

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -\frac{MU_1}{MU_2}$$

  where $MU_i = \frac{\partial u}{\partial x_i}$ is called the *marginal utility* of good i
- We usually have *diminishing marginal rates of substitution*

Examples

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- Preference relation: consumer is equally happy to swap one good for the other at rate of 1-to-1

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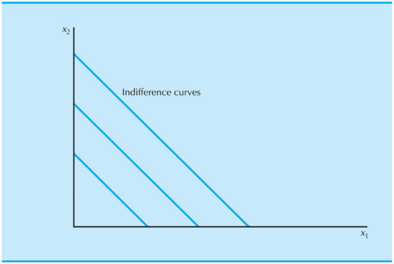

Figure 3.3

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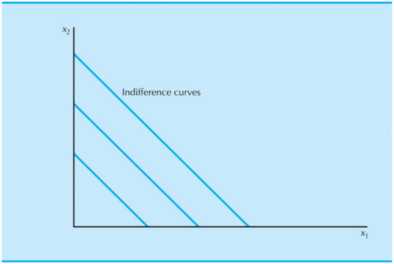

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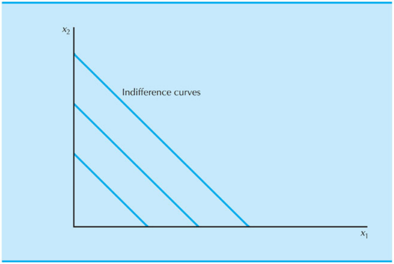

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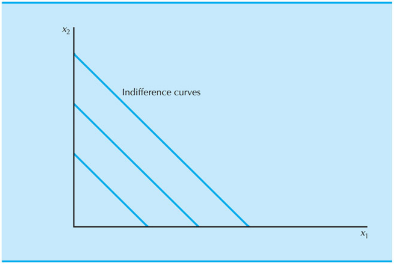

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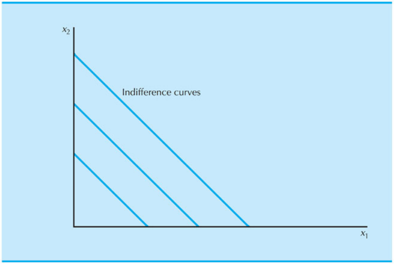

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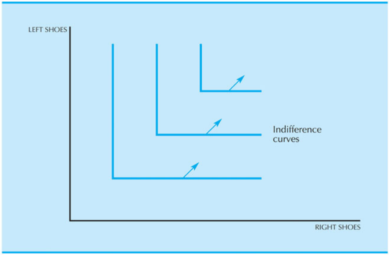

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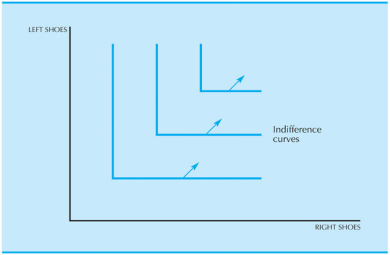

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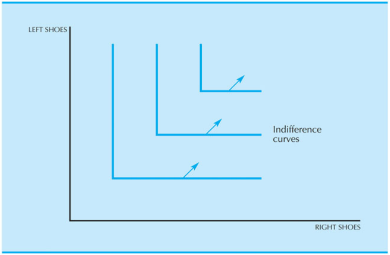

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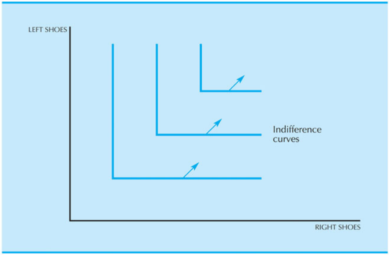

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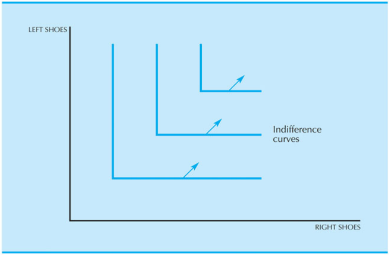

- Utility function: $u(x_1, x_2) = \min\{x_1, x_2\}$
- MRS = $-\infty$ or 0

Cobb-Douglas

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- MRS = $-\frac{c}{d}\frac{x_2}{x_1}$
- Very useful for approximating preferences in the real world