

# Econ 301: Microeconomic Analysis

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# Externalities

# Introduction

- ▶ Last time, saw that if all interaction between agents happened in a market, would obtain Pareto efficient outcome
- ▶ However, in general we have *externalities*, where agents have preferences for things that are not sold on the market
- ▶ Question for today: Will we still get Pareto efficient outcomes if there are goods with no market?

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# Consumption Externality Example: Smoking Roommate

- ▶ Let A and B be two people sharing a room
- ▶ A prefers to smoke while B prefers clean air
- ▶ A and B each endowed with some money:  $m_A, m_B$
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- ▶ Note that we can represent this situation with Edgeworth box:
  - ▶ Money on horizontal dimension, eg
  - ▶ Then vertical axis represents percent of smoky air for A, or conversely clean air for B
  - ▶ Total amount of air is fixed
  - ▶ Preferences for A increasing in money and in smoky air
  - ▶ Preferences for B increasing in money and in clean air

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- ▶ These are just two extremes; a whole continuum of possible property rights
- ▶ As long as property rights are clear and agreed upon, Pareto efficient allocation will be obtained

# Smoking Roommate Edgeworth Box

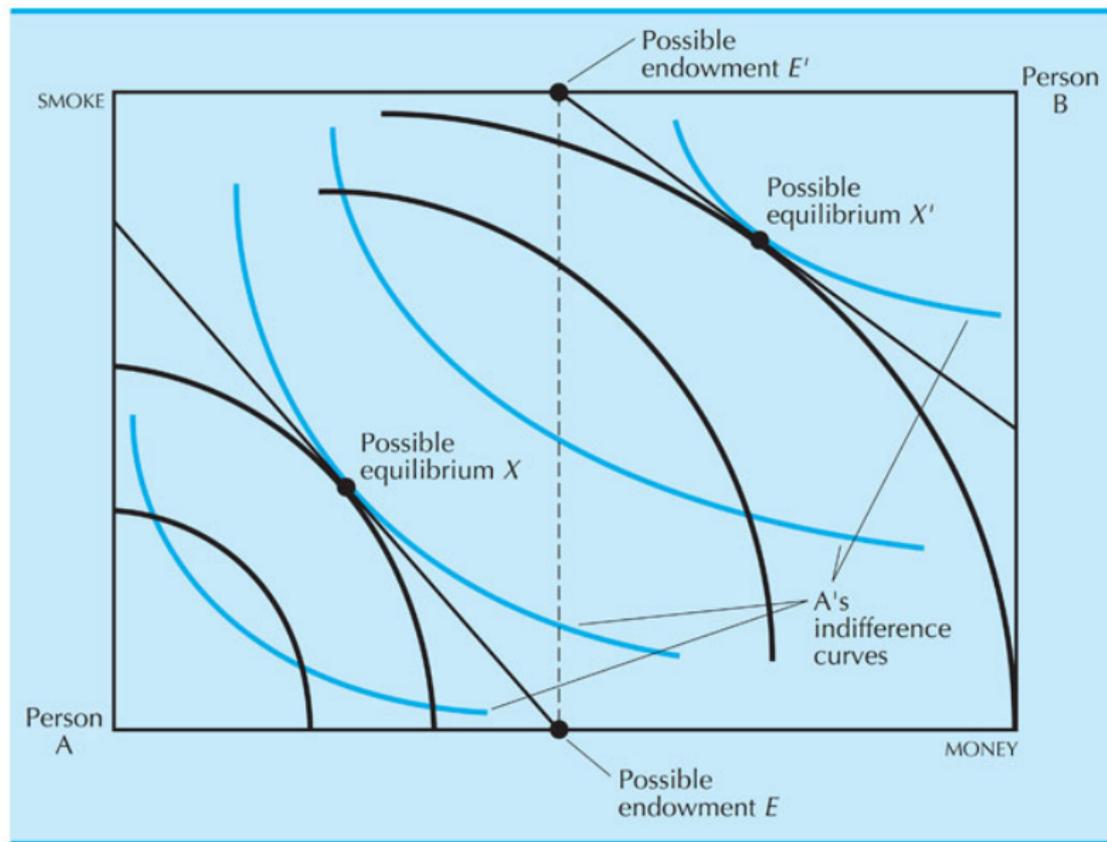


Figure 35.1

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  - ▶ So contract curve will be a straight horizontal line
- ▶ Whatever the initial endowment, agents end up consuming same amount of clean air
  - ▶ This is called *The Coase Theorem*
  - ▶ Big implication (if assumptions hold): initial property rights do not affect final allocations of clean/smoky air

# Coase Theorem Graphically

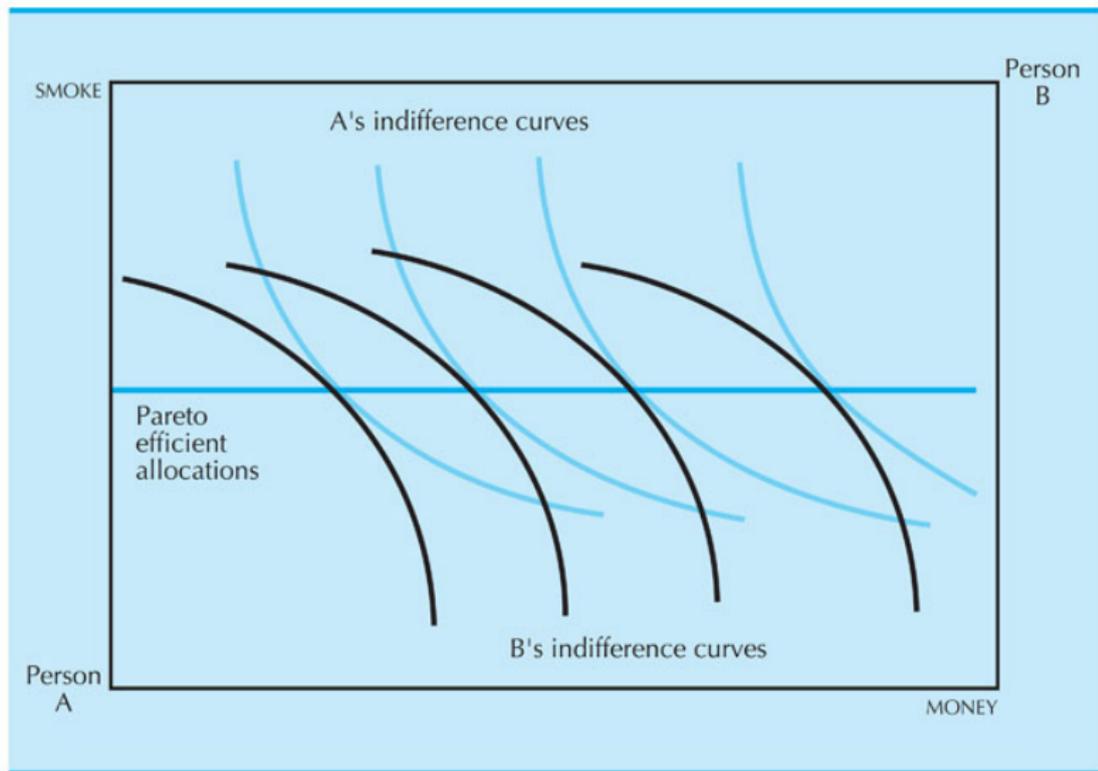


Figure  
35.2

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- ▶ Fishery downstream produces fish  $f$  at cost  $c_f(f, x)$ 
  - ▶ Fish sells at price  $p_f$
  - ▶ Note the steel mill can choose pollution amount  $x$  but fishery must take it as given
  - ▶ Assume  $\frac{dc_f}{dx} \geq 0$

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- ▶ FOC: just one (with respect to  $f$ ):

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- ▶ Note that first two FOC are same, but third FOC implies a lower level of pollution  $x$  that if firms were separate

# Terminology

- ▶ When the firms are acting independently, they are trying to minimize *private cost*
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- ▶ Note that if all costs are fully internalized, market equilibrium should give Pareto efficient outcome, as we expect from last lecture

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- ▶ Note: government has to know cost structures to set correct tax

# Market for Pollution

- ▶ Rather than government imposing pollution price through tax, we can add a market for pollution
- ▶ Then government just has to set property rights
- ▶ Assume fishery has right to clean water
- ▶ Then steel mill has to pay price  $q$  to fishery to pollute
- ▶ Fishery can sell pollution rights for price  $q$

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- ▶ Get same FOC as before, regardless of property rights!

# Tragedy of the Commons

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- ▶ Villagers' individual decisions
  - ▶ Suppose each villager can choose to buy a cow or not
  - ▶ Since number of villagers is relatively small, Nash Equilibrium is an appropriate tool here

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  - ▶ Thus private equilibrium number of cows is higher than socially optimal number of cows, leading to overgrazing
- ▶ What happened?
  - ▶ Unclear property rights lead villagers to graze more than their share
  - ▶ Solution: formalize property rights through regulation or ownership of commons

# Tragedy of the Commons Graphically

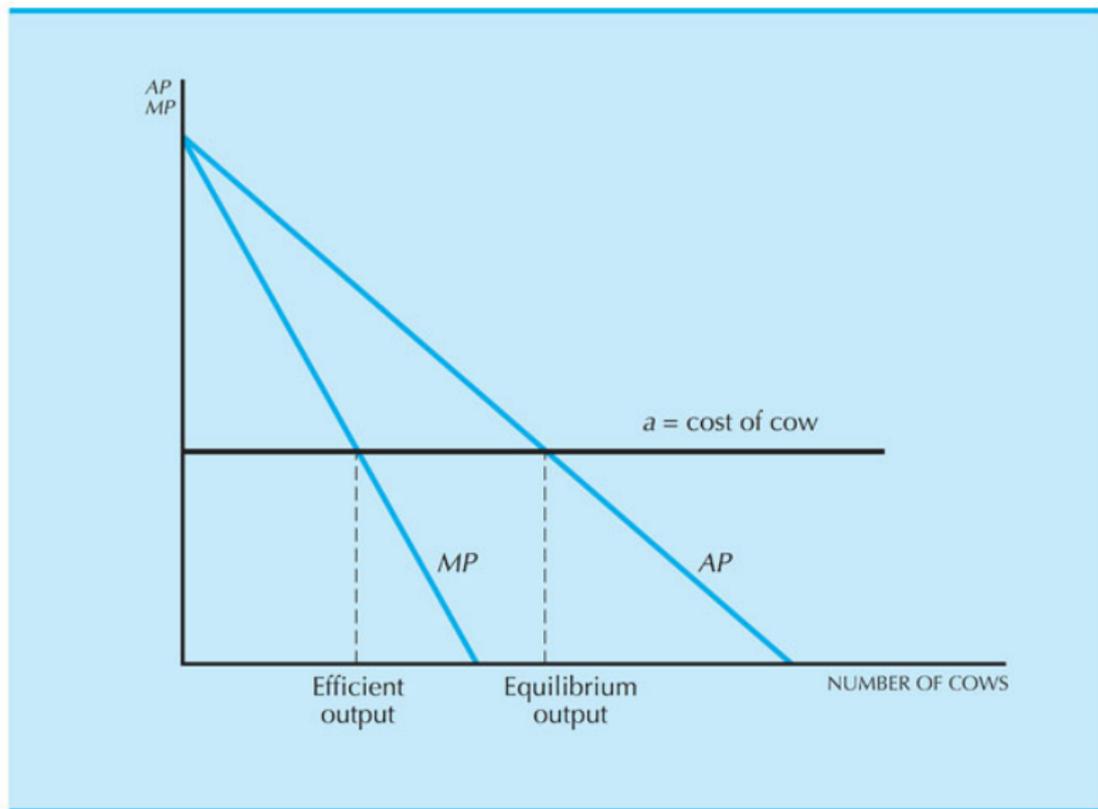


Figure 35.4