# Econ 301: Microeconomic Analysis

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# Revealed Preference

# Observing Choices

- So far in class: start with preferences/utility function, and derive choices
- Can we go the other way?
  - That is, can we derive preferences from observing choices?
- Two important assumptions throughout this section:
  1. Preferences are stable
  2. Strict convexity of preference (so unique maximum exists)

# Revealed Preference

- Suppose we observe $X = (x_1, x_2)$ chosen when $Y = (y_1, y_2)$ was available at prices $p_1, p_2$

**Definition**

If $X$ is chosen at prices $p_1, p_2$ and $p_1 y_1 + p_2 y_2 \leq p_1 x_1 + p_2 x_2$, then we say $X$ is *directly revealed preferred* to $Y$.

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**Theorem**

*If $X$ DRP $Y$, then $X \succ Y$.*

- That is, if we see $X$ chosen over $Y$, we should be able to infer that $X \succ Y$

# Chains of Revealed Preference

- What if we *also* observe $Y = (y_1, y_2)$ being chosen over $Z = (z_1, z_2)$ at different prices $q_1, q_2$?

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- Note $X \succ Y$ and $Y \succ Z$ from theorem, which implies $X \succ Z$ by transitivity

**Theorem**

*If $X$ IRP $Z$, then $X \succ Z$.*

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**Theorem**

*If $X$ IRP $Z$, then $X \succ Z$.*

**Definition**

If $X$ DRP $Z$ or $X$ IRP $Z$, we say $X$ is *revealed preferred* $Z$, or $X$ RP $Z$.

# Revealed Preference Graphically

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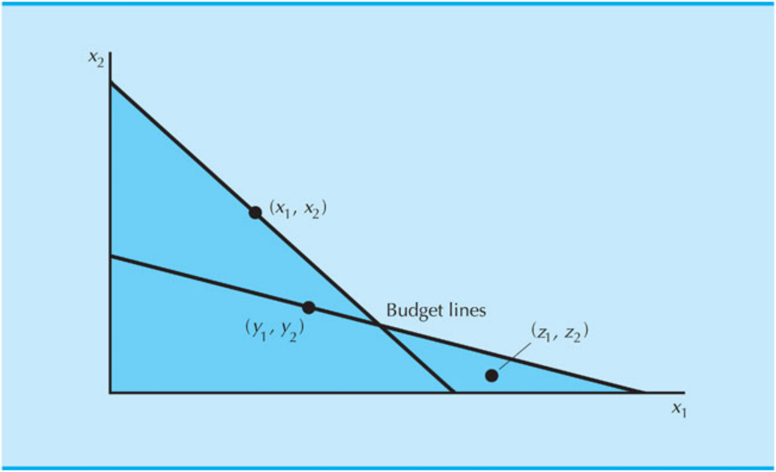


Figure 7.2

# What This Buys Us

- What do we get from setting up all these definitions?
- Two really nice tricks to go from choices to preferences:
  1. We can put bounds on indifference curves by observing choices
  2. We can check whether observed choices are consistent with maximizing behavior

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- If we further observe $Y$ DRP $X$ and $Z$ DRP $X$, which bundles can we compare to $X$?
  - Any bundle with at least as much good 1 and good 2 as $Y$ or $Z$ is strictly preferred to $X$, by monotonicity
  - Furthermore, any bundle with at least as much good 1 and good 2 than any *mixture* of $Y$ or $Z$with $X$ is strictly preferred to $X$, by convexity and monotonicity

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- In observing just 4 choices, we have put strong restrictions on where indifference curve through $X$ can lie

# Recovering Preferences Graphically

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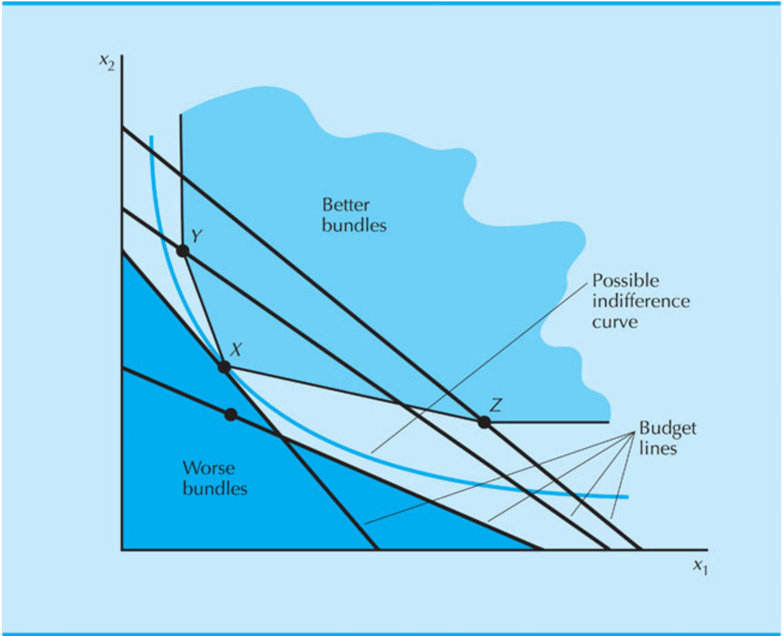


Figure 7.3

# Testing Choices: The Weak Axiom

- Suppose we observe $X$ DRP $Y$ from one budget set and $Y$ DRP $X$ from another budget set
  - Implies $X \succ Y$ and $Y \succ X$, a contradiction
  - Clearly such data would imply that consumer is not maximizing

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**Definition**

The *Weak Axiom of Revealed Preference (WARP)* states that if $X$ is directly revealed preferred to $Y$, then we cannot have $Y$ directly revealed preferred to $X$.

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**Definition**

The *Weak Axiom of Revealed Preference (WARP)* states that if $X$ is directly revealed preferred to $Y$, then we cannot have $Y$ directly revealed preferred to $X$.

**Theorem**

*If WARP is violated, then we can conclude that observed behavior is not consistent with maximizing model of consumer choice*

# Testing WARP

- Given data on prices and bundles chosen at those prices, how to we check for WARP violations?

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- Given data on prices and bundles chosen at those prices, how to we check for WARP violations?
- We use the following algorithm:
  - Calculate expenditure for each bundle at each possible price
    - Expenditure is $p_1 x_1 + p_2 x_2$
  - Put cost of bundle $i$ at price point $j$ in row $i$, column $j$ in matrix, ie cell $(i, j)$
  - Direct revealed preference when an off-diagonal entry is cheaper than then on-diagonal entry in the same column
  - Indicate direct revealed preference with $*$
  - If $*$ in at cell $(r, c)$ and $(c, r)$, violation of WARP

# Testing WARP: Example

- Suppose we observe the following three choices under the given prices:

| Choice Number | $p_1$ | $p_2$ | $x_1$ | $x_2$ |
|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 2 |
| 2 | 2 | 1 | 2 | 1 |
| 3 | 1 | 1 | 2 | 2 |

- Is there a violation of WARP?

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| Choice Number | Budget 1 | Budget 2 | Budget 3 |
|---|---|---|---|
| 1 | 5 | $4^*$ | $3^*$ |
| 2 | $4^*$ | 5 | $3^*$ |
| 3 | 6 | 6 | 4 |

- Violation: choice 1 DRP choice 2 and choice 2 DRP choice 1

# Testing Choices: The Strong Axiom

- The weak axiom works only one way:
  - If we have a violation, then consumer is not maximizing
  - But it we don't find a violation, we can't be sure if consumer is maximizing
  - That is, satisfying the weak axiom is necessary but not sufficient for maximizing behavior

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- The weak axiom works only one way:
  - If we have a violation, then consumer is not maximizing
  - But it we don't find a violation, we can't be sure if consumer is maximizing
  - That is, satisfying the weak axiom is necessary but not sufficient for maximizing behavior
- Luckily we have another condition that is both necessary and sufficient for maximizing behavior:

**Definition**

The *Strong Axiom of Revealed Preference (SARP)* states that if $X$ is revealed preferred (directly or indirectly) to $Y$, then $Y$ cannot be revealed preferred to $X$ (directly or indirectly).

# Index Numbers

# Quantity Indices

- Suppose we want to compare average purchasing behavior today (period $t$) to average purchasing behavior in some baseline year (period $b$)
- Needs weights $w_1$ and $w_2$ on goods 1 and 2:

$$I_q = \frac{w_1 x_1^t + w_2 x_2^t}{w_1 x_1^b + w_2 x_2^b}$$

- Weights tell us relative importance of the two good when evaluations how well off consumer is

# Paasche quantity index

- Let weights be today's prices
- This is called *Paasche quantity index*

$$P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$

- If $P_q > 1$, what can we say about how well off consumer is today vs baseline?

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  - Note $X^t$ DRP $X^b$, so consumer is better off today
- If $P_q < 1$, what can we say about how well off consumer is today vs baseline?
  - Can't say definitively

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# Price Indices

- Quantity indices compare average consumption in two periods
- What if we want to compare average prices?
  - Use price indices:

$$I_p = \frac{p_1^t w_1 + p_2^t w_2}{p_1^b w_1 + p_2^b w_2}$$

# Paasche Price Index

- Use today's consumption as weight:

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t}$$

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- Define relative change in income:

$$M = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

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  - Consumer is better off in base year because $X_b$ DRP $X_t$
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  - Can't say whether consumer is better of now or at baseline

# Laspeyres Price Index

- Use baseline period's consumption as weight:

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