Econ 301: Microeconomic Analysis

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Oliogopoly

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- Ideal tool to study the case where we have multiple firms interacting (so not monopoly) but firms are big enough to influence market price (so not pure competition)
  - This is *oligopoly*
- Two simplifying assumptions
  - Just two firms (*duopoly*)
  - Firms are producing identical products (no *product differentiation*)

Overview

- Two possible timings of firm choices
  - Sequential: use backwards induction
  - Simultaneous: use Nash equilibrium
- Two possible choice variables for firms:
  - Price
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- Two possible timings of firm choices
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- Two possible choice variables for firms:
  - Price
  - Quantity
- Thus there are four possible models:
  1. Sequential quantity competition (Stackleberg)
  2. Sequential price competition (won't cover this)
  3. Simultaneous quantity competition (Cournot)
  4. Simultaneous price competition (Bertrand)

Sequential Quantity Competition: Setup

- Firm 1 chooses quantity y_1 first (the *quantity leader* or *first mover*)
- Then firm 2 (the *follower* or *second mover*) chooses its quantity y_2
- Total quantity $Y = y_1 + y_2$
- Inverse demand $p(Y) = p(y_1 + y_2)$
- Cost functions $c_1(y_1)$ and $c_2(y_2)$
- Also known as *Stackleberg model*

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  - Note that firm 2's reaction function is in firm 1's problem
  - Solving this problem gives leader's quantity
  - Plug in to find follower's quantity

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- Then $y_2^* = \frac{a - b\frac{a}{2b}}{2b} = \frac{a}{4b}$
- Total quantity: $Y^* = y_1^* + y_2^* = \frac{3}{4}\frac{a}{b}$
- Note that there is *first mover advantage*

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Definition

The Nash equilibrium of the Cournot model (known as Cournot equilibrium) is a quantity pair (y_1^*, y_2^*) such that

$$y_1^* = f_1(y_2^*)$$
$$y_2^* = f_2(y_1^*)$$

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- Firm 2's problem is identical, so we know immediately that

$$y_2 = \frac{a - by_1}{2b}$$

- In equilibrium, have $y_1^* = y_2^*$ (only works when firms are identical)
- Thus we can solve to find $y_1^* = y_2^* = \frac{a}{3b}$

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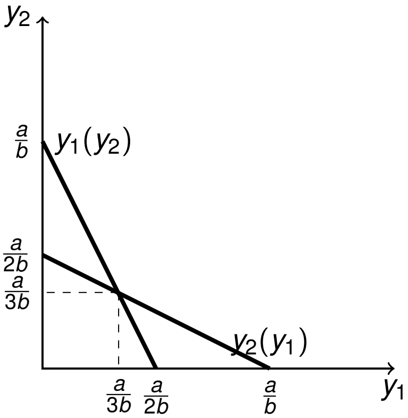

Simultaneous Price Setting

- Suppose instead firms simultaneously announce prices p_1 and p_2
- Both firms have constant marginal cost c
- Both firms have capacity to serve entire market
- Firm that announces lower price gets all of market share; if tie, they split market share
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- Only possibility: both firms announce $p_1 = p_2 = c$
- Note that this is the pure competitive equilibrium price!