Econ 301: Microeconomic Analysis

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Social Welfare

Motivation

- Pareto efficiency says very little about the distribution of resources between people
- In general there are many Pareto efficient outcomes in an economy
- Pareto efficiency is a minimum standard for an allocation, but can we be more exacting?

Aggregation of Preferences

- Let x stand for the allocation of all goods in to all agents
- Each agent i has preference ordering over allocations given by $\succ_i$
  - In general, agents care about outcome for everyone, not just themselves
  - Assume that $\succ_i$ is complete, reflexive, and transitive for all agents i
- We seek a *social preference ordering* $\succ_s$ that aggregates the preferences of all agents
- We'll need a *mechanism* that takes all the individual orderings $\{\succ_i\}_{i=1}^n$ and returns a social ordering $\succ_s$

Example

- Three individuals: A, B, C
- Three outcomes: x, y, z
- Preferences:
  - A: $x \succ_A y \succ_A z$
  - B: $y \succ_B z \succ_B x$
  - C: $z \succ_C x \succ_C y$
- Can represent preferences like this:

A	B	C
x	y	z
y	z	x
z	x	y

- What mechanisms could we use to get social ordering from individual preferences?

Majority Vote

- One possible mechanism: pairwise majority vote
  - $x \succ_s y$ if a majority of subjects prefer x to y
- Recall:

A	B	C
x	y	z
y	z	x
z	x	y

- Will the resulting social preference relation be transitive? No:
  - $x \succ_s y$ by a 2-to-1 vote
  - $y \succ_s z$ by a 2-to-1 vote
  - $z \succ_s x$ by a 2-to-1 vote, but transitivity would require $x \succ_s z$
- Note: by picking the order of pairwise voting we can force any option to by picked from the full set (x, y, z)

Borda Count

- Another possible mechanism: let people report their rankings (1, 2, 3, ect) of each option
- Add up the rankings for each option
- Then $x \succ_s y$ if the aggregate ranking of x is lower than that of y
- Example:

A	B
x	y
y	z
z	x

- Suppose only x and y put on the table; which one wins?
  - Both get 3 total points, so $x \sim_s y$ (a tie)
- What if all three options put on the table?
  - $y \succ_s x \succ_s z$ (3 points to 4 points to 5 points)
- So adding z to the set of options changes the social preference ordering of x and y

Desirable Properties of Social Preference Orderings

1. If individual preferences are complete, reflexive, and transitive, then the social preference ordering should be as well.
    - Majority vote violates this
2. The social preference ordering between x and y should only depend on the how individuals rank x vs y.
    - Property sometimes called *independence of irrelevant alternatives (IIA)*
    - Borda count violates this
3. If $x \succ_i y$ for all individuals, then $x \succ_s y$
4. No dictatorship. A *dictatorship* is when there is some individual i such that $x \succ_s y$ if and only if $x \succ_i y$.

Arrow's Impossibility Theorem

Theorem (Arrow)

There is no mechanism that satisfies properties 1 through 4.

Theorem (Arrow, alternate version)

The only mechanism that satisfies properties 1 through 3 is a dictatorship.

- So what do we do?
  - If we drop property 2 (IIA), many voting mechanisms will satisfy remaining properties (such as Borda count)

Social Welfare Functions

- Instead of aggregating preference rankings, we can aggregate utility functions
- Define a *social welfare function* as

$$W(x) = W\left(u_1(x), u_2(x), \ldots, u_n(x)\right)$$

  where n is number of individuals in economy
  - Only requirement is that W is increasing in each argument
  - Issue: utility functions are not unique, so we have to pick one representation
- Once we have picked social welfare function:

$$x \succ_s y \text{ if and only if } W(x) > W(y)$$

Example Social Welfare Functions

- Social welfare function can take many forms
- Possible social welfare functions
  - Classical:
    $$W(x) = \sum_{i=1}^{n} u_i(x)$$
  - Weighted sum:
    $$W(x) = \sum_{i=1}^{n} \alpha_i u_i(x)$$
  - Rawlsian:
    $$W(x) = \min\{u_1(x), u_2(x), \ldots, u_n(x)\}$$

Social Welfare Maximization

- Let there be k goods and n individuals in economy
- Assume there are amounts $X^1, \dots, X^k$ of each good available via endowments
- Then the *social welfare maximization problem* or *social planner's problem* is

$$\max_x W(x) \text{ s.t. } \sum_{i=1}^{n} x_i^j = X^j \text{ for all } j \in \{1, 2, \dots k\}$$

- Call solution a *maximal welfare allocation*

Visualizing Social Welfare Maximization

- For simplicity assume 2 agents
- Make axes u_1 and u_2
- *Utility possibilities set*: set of combinations of utility for agents 1 and 2 that are feasible given endowments
- Edge of this set is *utility possibilities frontier*
  - What is special about points on fronteir? They are Pareto efficient
- *Isowelfare curves* trace out points in space that give same social welfare
  - What determines shape of isowelfare curves? Social welfare function
- Maximal welfare allocation is at point on utility possibility frontier that reaches highest isowelfare curve

Social Welfare Maximization

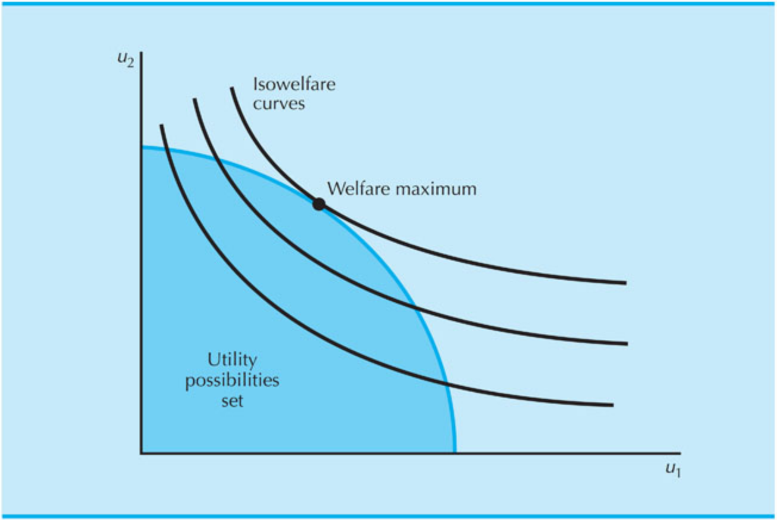

Figure 34.1

Welfare Maximization and Pareto Efficiency

- Will a maximal welfare allocation be Pareto efficient? Yes.
  - Maximal welfare allocation must be on utility possibilities frontier
- Will a Pareto efficient allocation be a maximal welfare allocation (for some welfare function)? Yes.
  - By changing weights on weighted social welfare function, can make tangency at any point desired
  - That is, a social welfare function exists that can justify any Pareto efficient allocation

Symmetric Allocations

- Consider an allocation that is *symmetric*, meaning that each agent gets equal amounts of each good
- Will this be Pareto efficient?
- Consider following example:
  - Three agents: A, B, C
  - Two goods: x_1 and x_2, one unit of each in economy
  - Agents A and B care only about having more of good 1
  - Agent C cares only about having more of good 2
- Symmetric allocation?
  - Each get $\frac{1}{3}$ units of good 1 and $\frac{1}{3}$ units of good 2
- Is this Pareto optimal?
  - A and C would like to trade, so that A gets $\frac{2}{3}$ units of good 1 and C gets $\frac{2}{3}$ units of good 2
  - Therefore symmetric allocation is not Pareto efficient in this case (and thus not welfare-maximixing)