Econ 301: Microeconomic Analysis

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Intertemporal Choice

Motivation

- Much interesting economic behavior happens over many time periods
- Examples?
  - Retirement savings
  - Long-term investments by businesses
  - College savings (and returns to college education)
  - Stock market choices
- We need a way to formally analyze these types of choices
  - For simplicity, assume small number of discrete time periods
  - Preferences do not change period-to-period, but relative impact of one period's consumption depends on how far in the future it is

Setup

- Two time periods: $t = 1, 2$
- A single good consumed in either period
  - Consumption c_1 and c_2 in periods 1 and 2, respectively
  - For now, assume price of consumption good is 1 in both periods
- Endowment $E = (m_1, m_2)$ in dollars
- Interest rate r
  - \$1 of endowment not spent in period 1 grows becomes \$$(1 + r)$ in period 2

Deriving the Budget Constraint

- Suppose you consume $c_1 < m_1$ in period 1
  - How much money stays in your bank account? $m_1 - c_1$
  - How much can you consume in period 2?
    $c_2 = (1 + r)(m_1 - c_1) + m_2$
  - Rearrange: $(1 + r)c_1 + c_2 = (1 + r)m_1 + m_2$
- Suppose you consume $c_1 > m_1$ in period 1
  - Instead of saving, borrow at interest rate r
  - How much money do you borrow? $c_1 - m_1$
  - How much do you have to pay back in period 2? $(1 + r)(c_1 - m_1)$
  - How much can you consume in period 2?
    $c_2 = m_2 - (1 + r)(c_1 - m_1)$
  - Rearrange: $(1 + r)c_1 + c_2 = (1 + r)m_1 + m_2$
- Note same budget constraint whether save or borrow!

The Intertemporal Budget Constraint

- The budget constraint (*future value* formulation):

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

- The budget constraint another way (*present value* formulation):

$$c_1 + \frac{1}{1+r}c_2 = m_1 + \frac{1}{1+r}m_2$$

- These are the exact same formula
  - Just divide FV by $(1+r)$ to get PV
- Why we have two ways of writing budget will be clear shortly

Drawing the Budget Constraint

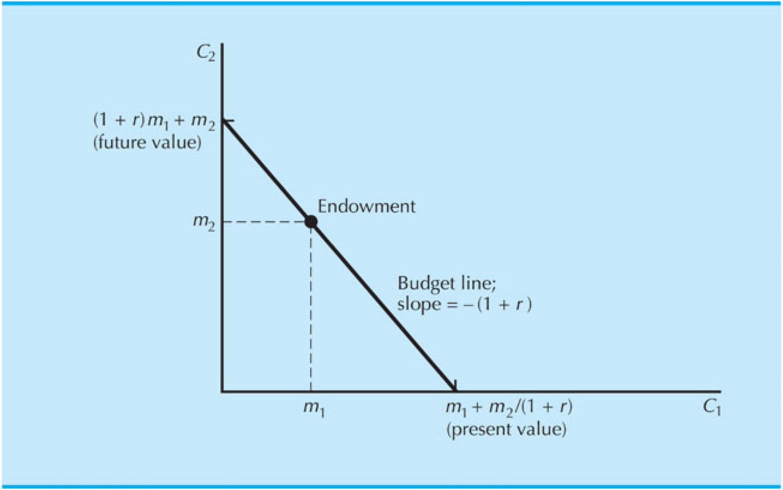


Figure 10.2

Future Value vs Present Value

- What is value of $1 today in terms of consumption tomorrow?
  - Recall $(1+r)c_1 + c_2 = (1+r)m_1 + m_2$
  - One extra dollar today can buy $1+r$ units of consumption tomorrow
  - Thus *future value* of $1 is $\$(1+r)$
- What is value of $1 in the future in terms of consumption today?
  - Recall $c_1 + \frac{1}{(1+r)}c_2 = m_1 + \frac{1}{(1+r)}m_2$
  - One extra dollar in the future can buy $\frac{1}{(1+r)}$ units of consumption today
  - Thus *present value* of $1 is $\$\frac{1}{(1+r)}$
- By convention, we typically work with present value (PV)
- Note budget constraint says PV(endowment) = PV(consumption)
- Higher PV means the consumer can consume more in every period

Higher PV Makes Consumer Better Off

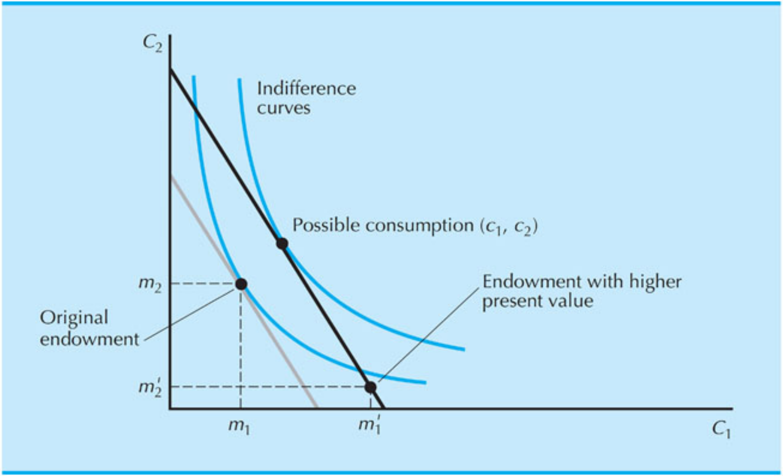


Figure 10.6

Borrowing and Lending

- If $c_1 > m_1$, you are a *net borrower*
- If $c_1 < m_1$, you are a *net lender*

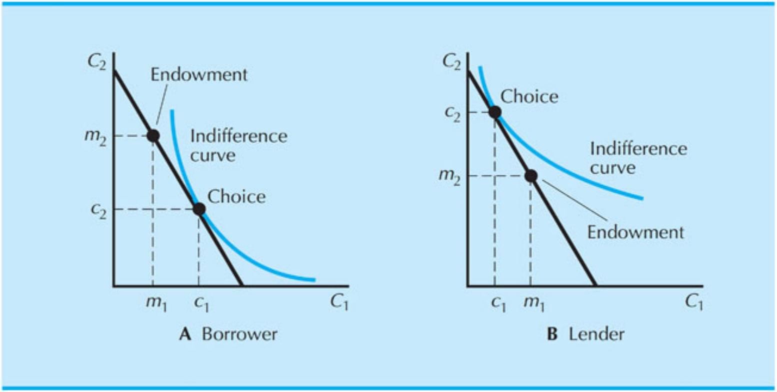


Figure 10.3

Effect of Increasing Interest Rate

- What happens when interest rate r increases?
  - Budget line becomes steeper
  - Budget line rotates around endowment point
  - What do borrowers do? Some become lenders
  - What do lenders do? Stay lenders

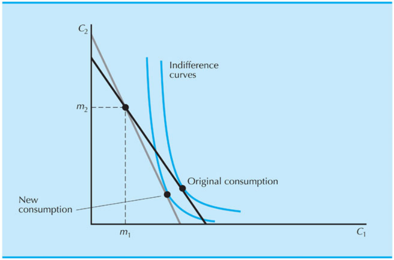
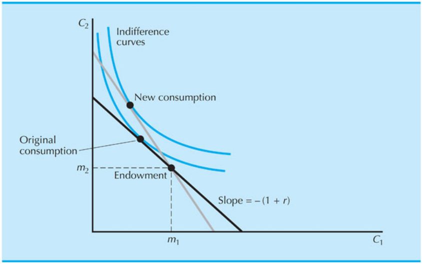


Figure 10.5

Figure 10.4



Effect of Decreasing Interest Rate

- What happens when interest rate r decreases?
  - Budget line becomes shallower
  - Budget line rotates around endowment point
  - What do borrowers do? Stay borrowers
  - What do lenders do? Some become borrowers

Intertemporal Utility

- Utility is $U(c_1, c_2) = u(c_1) + \frac{1}{1+\delta} u(c_2)$
- $\delta \geq 0$ is *discount factor*
- Non-zero δ means future consumption less valuable than today's
- The higher the δ, the less patient decision-maker is
- For example, suppose $u(c) = c$ and $\delta = 0.2$
  - Would DM rather have $(c_1, c_2) = (0, 12)$ or $(9, 0)$?
  - $U(9, 0) = 9 < U(0, 12) = \frac{1}{1.2} 12 = 10$, so prefers $(0, 12)$
- But suppose $\delta = 0.5$ (more impatient)
  - Would DM rather have $(c_1, c_2) = (0, 12)$ or $(9, 0)$?
  - $U(9, 0) = 9 > U(0, 12) = \frac{1}{1.5} 12 = 8$, so prefers $(9, 0)$
  - That is, DM gives up 3 units to get consumption earlier

Intertemporal Choices

- What is general maximization problem?

$$max_{c_1,c_2} u(c_1) + \frac{1}{1+\delta} u(c_2) \text{ s.t. } c_1 + \frac{1}{1+r} c_2 = m_1 + \frac{1}{1+r} m_2$$

- What is solution?
- Set up Lagrangian:

$$\mathcal{L} = u(c_1) + \frac{1}{1+\delta} u(c_2) + \lambda \left(m_1 + \frac{1}{1+r} m_2 - c_1 - \frac{1}{1+r} c_2 \right)$$

- FOC:

$$\frac{\partial \mathcal{L}}{\partial c_1} = 0 \rightarrow \qquad u'(c_1) = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = 0 \rightarrow \qquad u'(c_2) = \lambda \tfrac{1+\delta}{1+r}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \rightarrow \quad \text{budget constraint}$$

Intertemporal Choices

- Take the ratio of the FOC:

$$\frac{u'(c_1)}{u'(c_2)} = \frac{1+r}{1+\delta}$$

- Assume that $\frac{\partial u}{\partial c} > 0$ and $\frac{\partial^2 u}{\partial c^2} < 0$
- What happens if $\delta = r$?

$$u'(c_1) = u'(c_2) \rightarrow c_1^* = c_2^*$$

- What happens if $\delta < r$?

$$u'(c_1) > u'(c_2) \rightarrow c_1^* < c_2^*$$

Allowing for Inflation

- What if different prices in each period: p_1 and p_2?
- Assume endowment m is in consumption units
- Suppose you consume c_1 in period 1
- How much money stays in your bank account? $p_1 m_1 - p_1 c_1$
- How much can you consume in period 2?

$$c_2 = \frac{1}{p_2}\left[(1+r)(p_1 m_1 - p_1 c_1) + p_2 m_2\right]$$

- Rearrange to find FV formulation of budget constraint:

$$(1+r)p_1 c_1 + p_2 c_2 = (1+r)p_1 m_1 + p_2 m_2$$

- Divide through by $(1+r)$ to get PV formulation:

$$p_1 c_1 + \frac{1}{1+r} p_2 c_2 = p_1 m_1 + \frac{1}{1+r} p_2 m_2$$

Inflation and Real Interest Rates

- Let $\frac{p_2}{p_1} = 1 + \pi$, where π is the *inflation rate*
- Then PV formulation of budget becomes

$$c_1 + \frac{1+\pi}{1+r} c_2 = m_1 + \frac{1+\pi}{1+r} m_2$$

- Let $\frac{1+r}{1+\pi} = 1 + \rho$, where ρ is the *real interest rate*:

$$c_1 + \frac{1}{1+\rho} c_2 = m_1 + \frac{1}{1+\rho} m_2$$

- Looks just like non-inflation BC with r replaced by ρ
- Note: can also do BC with dollar-valued m's instead of consumption-valued m's

Multiple Periods

- Suppose we have 3 periods (with same price)
- Budget constraint if can borrow or lend at rate r between two consecutive periods

$$c_1 + \frac{1}{1+r}c_2 + \frac{1}{(1+r)^2}c_3 = m_1 + \frac{1}{1+r}m_2 + \frac{1}{(1+r)^2}m_3$$

- What if interest rate changes each period?
  - Eg r_1 between periods 1 and 2, r_2 between periods 2 and 3

$$c_1+\frac{1}{1+r_1}c_2+\frac{1}{(1+r_1)(1+r_2)}c_3 = m_1+\frac{1}{1+r_1}m_2+\frac{1}{(1+r_1)(1+r_2)}m_3$$

Net Present Value

- When choosing between two investment opportunities, choose one with higher present value
- May need to pay P_1 and P_2 to get income M_1 and M_2 in periods 1 and 2, respectively
- In this case, compare net income flows, ie *net present value*:

$$NPV = (M_1 - P_1) + \frac{1}{1+r}(M_2 - P_2)$$

Example: Bonds

- Financial securities: instruments that pay out money over time
- Bonds are one type of security
  - Typically a way for governments or companies to borrow from consumers
  - Bond holder (consumer) pays for bond up front
  - Bond issuer pays *coupon* x every period until period T plus *face value* F in period T (*maturity date*)
- What is present value of a bond?
  - Income stream is $(x, x, x, \dots, x, F)$
  - $PV = \frac{1}{1+r}x + \frac{1}{(1+r)^2}x + \dots + \frac{1}{(1+r)^{T-1}}F$
- What is price of a bond?
  - Same as present value, so that NPV is zero