

Econ 301: Microeconomic Analysis

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Social Welfare

Motivation

- ▶ Pareto efficiency says very little about the distribution of resources between people
- ▶ In general there are many Pareto efficient outcomes in an economy
- ▶ Pareto efficiency is a minimum standard for an allocation, but can we be more exacting?

Aggregation of Preferences

- ▶ Let x stand for the allocation of all goods in to all agents
- ▶ Each agent i has preference ordering over allocations given by \succsim_i
 - ▶ In general, agents care about outcome for everyone, not just themselves
 - ▶ Assume that \succsim_i is complete, reflexive, and transitive for all agents i
- ▶ We seek a *social preference ordering* \succsim_s that aggregates the preferences of all agents
- ▶ We'll need a *mechanism* that takes all the individual orderings $\{\succsim_i\}_{i=1}^n$ and returns a social ordering \succsim_s

Example

- ▶ Three individuals: A, B, C
- ▶ Three outcomes: x, y, z
- ▶ Preferences:
 - ▶ $A: x \succ_A y \succ_A z$
 - ▶ $B: y \succ_B z \succ_B x$
 - ▶ $C: z \succ_C x \succ_C y$

- ▶ Can represent preferences like this:

A	B	C
x	y	z
y	z	x
z	x	y

- ▶ What mechanisms could we use to get social ordering from individual preferences?

Majority Vote

- ▶ One possible mechanism: pairwise majority vote
 - ▶ $x \succ_s y$ if a majority of subjects prefer x to y

- ▶ Recall:

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 - ▶ $x \succ_s y$ by a 2-to-1 vote
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- ▶ Note: by picking the order of pairwise voting we can force any option to be picked from the full set (x, y, z)

Borda Count

- ▶ Another possible mechanism: let people report their rankings (1, 2, 3, ect) of each option
- ▶ Add up the rankings for each option
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- ▶ So adding z to the set of options changes the social preference ordering of x and y

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3. If $x \succ_i y$ for all individuals, then $x \succ_s y$
4. No dictatorship. A *dictatorship* is when there is some individual i such that $x \succ_s y$ if and only if $x \succ_i y$.

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- ▶ So what do we do?
 - ▶ If we drop property 2 (IIA), many voting mechanisms will satisfy remaining properties (such as Borda count)

Social Welfare Functions

- ▶ Instead of aggregating preference rankings, we can aggregate utility functions
- ▶ Define a *social welfare function* as

$$W(x) = W(u_1(x), u_2(x), \dots, u_n(x))$$

where n is number of individuals in economy

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- ▶ Once we have picked social welfare function:

$$x \succ_s y \text{ if and only if } W(x) > W(y)$$

Example Social Welfare Functions

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- ▶ Possible social welfare functions
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- ▶ Rawlsian:

$$W(x) = \min\{u_1(x), u_2(x), \dots, u_n(x)\}$$

Social Welfare Maximization

- ▶ Let there be k goods and n individuals in economy
- ▶ Assume there are amounts X^1, \dots, X^k of each good available via endowments
- ▶ Then the *social welfare maximization problem* or *social planner's problem* is

$$\max_x W(x) \text{ s.t. } \sum_{i=1}^n x_i^j = X^j \text{ for all } j \in \{1, 2, \dots, k\}$$

- ▶ Call solution a *maximal welfare allocation*

Visualizing Social Welfare Maximization

- ▶ For simplicity assume 2 agents
- ▶ Make axes u_1 and u_2
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- ▶ *Isowelfare curves* trace out points in space that give same social welfare
 - ▶ What determines shape of isowelfare curves? Social welfare function
- ▶ Maximal welfare allocation is at point on utility possibility frontier that reaches highest isowelfare curve

Social Welfare Maximization

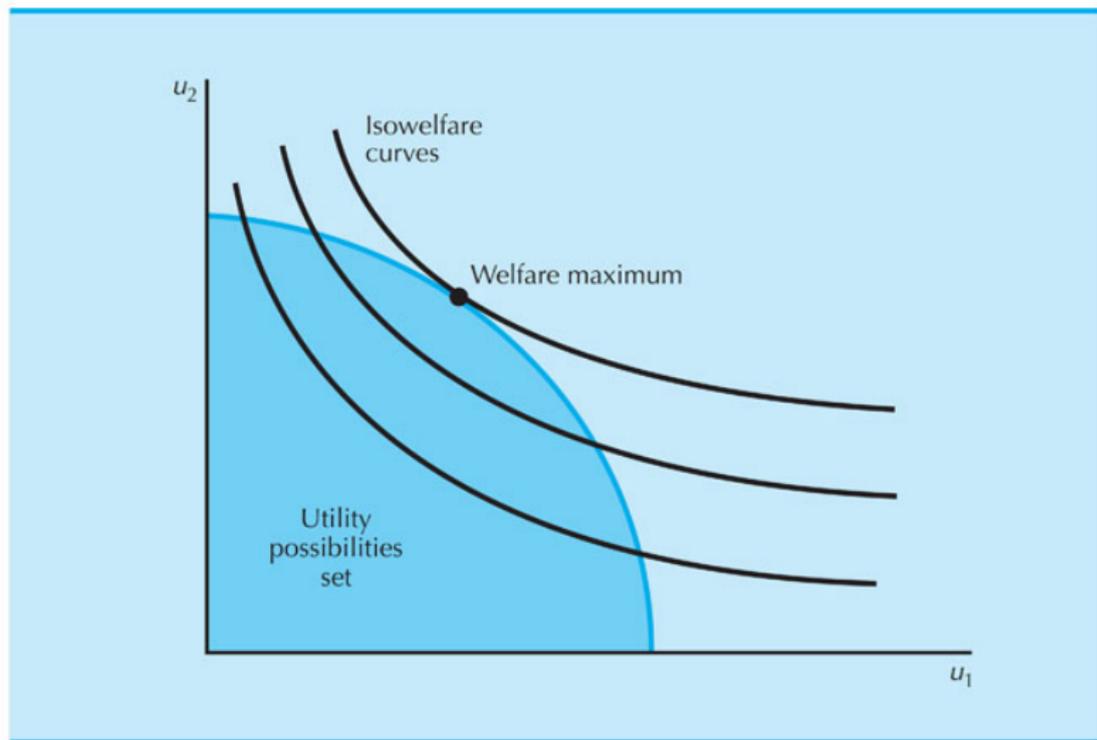


Figure
34.1

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 - ▶ Maximal welfare allocation must be on utility possibilities frontier
- ▶ Will a Pareto efficient allocation be a maximal welfare allocation (for some welfare function)? Yes.
 - ▶ By changing weights on weighted social welfare function, can make tangency at any point desired
 - ▶ That is, a social welfare function exists that can justify any Pareto efficient allocation

Symmetric Allocations

- ▶ Consider an allocation that is *symmetric*, meaning that each agent gets equal amounts of each good
- ▶ Will this be Pareto efficient?
- ▶ Consider following example:
 - ▶ Three agents: A , B , C
 - ▶ Two goods: x_1 and x_2 , one unit of each in economy
 - ▶ Agents A and B care only about having more of good 1
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 - ▶ Each get $\frac{1}{3}$ units of good 1 and $\frac{1}{3}$ units of good 2
- ▶ Is this Pareto optimal?
 - ▶ A and C would like to trade, so that A gets $\frac{2}{3}$ units of good 1 and C gets $\frac{2}{3}$ units of good 2
 - ▶ Therefore symmetric allocation is not Pareto efficient in this case (and thus not welfare-maximizing)