

### Motivation

- ▶ So far in this class we have looked at one market at a time
  - ▶ Equilibrium in just one market (ignoring all others) is called *partial equilibrium*
- ▶ But in general what happens in one market will affect outcomes in other markets
  - ▶ So we move to study *general equilibrium*, which is equilibrium of all markets in the economy at the same time
- ▶ Simplifying assumptions:
  - ▶ Fully competitive markets
  - ▶ Just two markets and two consumers
  - ▶ Focus on *pure exchange* for now: trade with no production

### Edgeworth Box: Setup

- ▶ We need to add a tool to our toolbox to tackle this problem
- ▶ Suppose two consumers, *A* and *B* and two goods, 1 and 2
- ▶ Consumers have initial *endowments*  $\omega_A = (\omega_A^1, \omega_A^2)$  and  $\omega_B = (\omega_B^1, \omega_B^2)$
- ▶ Consumers demand or *allocations* are  $x_A = (x_A^1, x_A^2)$  and  $x_B = (x_B^1, x_B^2)$
- ▶ An allocation  $(x_A, x_B)$  is *feasible* if  $x_A^1 + x_B^1 = \omega_A^1 + \omega_B^1$  and  $x_A^2 + x_B^2 = \omega_A^2 + \omega_B^2$

## Drawing the Edgeworth Box

- ▶ Width of box: total amount of good 1 in economy:  $\omega_A^1 + \omega_B^1$
- ▶ Height of box: total amount of good 2 in economy:  $\omega_A^2 + \omega_B^2$
- ▶ Endowment  $W = (\omega_A, \omega_B)$  is a point in the box
- ▶ Consumer A's allocation measured from lower left corner, while consumer B's endowment measured from upper right corner
- ▶ Consumer A's indifference curves open up and to the right, while consumer B's indifference curves open down and to the left

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## Edgeworth Box

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## Trade in the Edgeworth Box

- ▶ Suppose that the consumers start at a point  $W = (\omega_A, \omega_B)$  in the box
- ▶ Remember, an allocation is *Pareto efficient* if no one can be made better off without making someone worse off
- ▶ Is the endowment point is Pareto efficient?

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## Pareto Efficient Allocations in the Box

- ▶ Are there any allocations that are Pareto efficient?
- ▶ Is there more than one such point?
- ▶ Thus any trade starting at the endowment must end up on the contract curve and inside the lens
  - ▶ We call this part of the contract curve the *core*

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## Contract Curve

## Adding Prices

- ▶ Imagine a neutral third party (often called the *auctioneer*) who sets prices  $p = (p_1, p_2)$  for goods 1 and 2
- ▶ Based on preferences and budget, we can calculate each consumer's demand (sometimes called *gross demand*):

$$x_A = x_A(p, m_A) = (x_A^1(p, m_A), x_A^2(p, m_A))$$

$$x_B = x_B(p, m_B) = (x_B^1(p, m_B), x_B^2(p, m_B))$$

- ▶ We then define *excess* or *net demand* for each consumer:

$$e_A = (e_A^1, e_A^2) = (x_A^1 - \omega_A^1, x_A^2 - \omega_A^2)$$

$$e_B = (e_B^1, e_B^2) = (x_B^1 - \omega_B^1, x_B^2 - \omega_B^2)$$

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## The Budget Constraint

## Demand in the Edgeworth Box

- ▶ Budget constraints for the two consumers are represented by the *same line* in the Edgeworth box

- ▶ Note that the endowment point is on the budget set implied by the prices: consumer could decide to just consume their endowment

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## Competitive Equilibrium

- ▶ The economy is in *competitive equilibrium* (or *Walrasian equilibrium*) at prices  $p^* = (p_1^*, p_2^*)$  and endowment  $W = (\omega_A, \omega_B)$  when total demand equals total supply in each market

- ▶ For 2-good economy, this means we have

$$x_A^1(p^*) + x_B^1(p^*) = \omega_A^1 + \omega_B^1$$

$$x_A^2(p^*) + x_B^2(p^*) = \omega_A^2 + \omega_B^2$$

- ▶ This is called *market clearing* condition
- ▶ Note that since both consumers are optimizing, their indifference curves must be tangent to budget curve
  - ▶ And since they face the same prices, the indifference curves must also be tangent to each other
- ▶ Equilibrium is guaranteed to exist (as long as each consumer's demand is continuous, or each consumer is small relative to the market)

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## Aggregate Excess Demand

- ▶ We can define the *aggregate excess demand* for each good

$$z_1(p) = \underbrace{x_A^1(p) - \omega_A^1}_{e_A^1(p)} + \underbrace{x_B^1(p) - \omega_B^1}_{e_B^1(p)}$$

$$z_2(p) = \underbrace{x_A^2(p) - \omega_A^2}_{e_A^2(p)} + \underbrace{x_B^2(p) - \omega_B^2}_{e_B^2(p)}$$

- ▶ This gives us a new definition of competitive equilibrium:

$$z_1(p^*) = 0$$

$$z_2(p^*) = 0$$

- ▶ That is, aggregate excess demand of each good must be zero
- ▶ Each consumer wants to buy exactly as much as the other is selling (or vice versa)

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## Walras's Law

- ▶ Note consumer A's budget constraint can be written as

$$p_1 x_A^1 + p_2 x_A^2 = p_1 \omega_A^1 + p_2 \omega_A^2$$

- ▶ Rearranging, we get  $p_1(x_A^1 - \omega_A^1) + p_2(x_A^2 - \omega_A^2) = 0$ , or equivalently

$$p_1 e_A^1 + p_2 e_A^2 = 0$$

- ▶ Similarly, for B we will get  $p_1 e_B^1 + p_2 e_B^2 = 0$
- ▶ Adding A and B's conditions together give

$$p_1(e_A^1 + e_B^1) + p_2(e_A^2 + e_B^2) = 0$$

or

$$p_1 z_1 + p_2 z_2 = 0$$

- ▶ This last expression is known as *Walras's Law*

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## Using Walras's Law

- ▶ Note that if  $z_1 = 0$  then Walras's Law gives  $p_2 z_2 = 0$
- ▶ As long as  $p_2 > 0$  this implies  $z_2 = 0$  as well
- ▶ Clearly by same logic if  $z_2 = 0$  we can immediately conclude that  $z_1 = 0$  (as long as  $p_1 > 0$ )
- ▶ This means that for equilibrium it is sufficient to check just one of the two excess demand conditions
- ▶ In general, if we have  $k$  goods and  $k - 1$  of them are in equilibrium, the  $k$ th market will be in equilibrium as well
- ▶ Note that we are therefore free to set price of one good equal to 1 (the *numeraire good*)

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## Equilibrium in the Edgeworth Box

## Equilibrium Example

- ▶ Suppose both consumer have Cobb-Douglas preferences:

$$u_A(x_A^1, x_A^2) = (x_A^1)^a (x_A^2)^{1-a}$$

$$u_B(x_B^1, x_B^2) = (x_B^1)^b (x_B^2)^{1-b}$$

- ▶ What is (gross) demand for the two consumers?

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## Equilibrium Example (continued)

## Contract Curve

- ▶ What are equilibrium prices  $p_1^*$  and  $p_2^*$ ?

- ▶ What is formula for contract curve in this example?

- ▶ Setting  $p_2^* = 1$  (numeraire) and solving for  $p_1$  we get

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## First Welfare Theorem

- ▶ Will a competitive equilibrium be Pareto efficient?
  - ▶ Suppose equilibrium  $(x_A, x_B, p_1, p_2)$  was not Pareto efficient
  - ▶ Then there must exist allocation  $(y_A, y_B)$  that is both feasible and desirable for both consumers:
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- ▶ For  $(x_A, x_B)$  to be optimal it must be that  $(y_A, y_B)$  was not affordable:

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## First Welfare Theorem (continued)

- ▶ Adding these last two equations together we get
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- ▶ Plugging in the feasibility condition we get
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- ▶ Clearly a contradiction
  - ▶ Thus it must be that any competitive equilibrium is Pareto efficient
    - ▶ This is known as the *First Welfare Theorem*
  - ▶ Huge implication: Market process will automatically find an efficient outcome (though not necessarily a fair one)

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## Second Welfare Theorem

- ▶ OK, so the First Welfare Theorem says that a competitive equilibrium is Pareto efficient
- ▶ Is the converse true? That is, are all Pareto efficient allocations possible equilibria?
- ▶ Yes, any Pareto efficient allocation can be a competitive equilibrium for some prices  $p$  and endowments  $W$ 
  - ▶ This is the *Second Welfare Theorem*
  - ▶ Guaranteed as long as preferences are convex
  - ▶ Intuition: for Pareto efficiency, indifference curves are tangent, so we can find prices and endowment to run a budget curve right through the tangency point
- ▶ Huge implication: To get a desired efficient market outcome, just have to choose starting endowment and let market forces do their work

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