

Econ 301: Microeconomic Analysis

Prof. Jeffrey Naecker

Wesleyan University

Consumer Surplus

1 / 26

2 / 26

Motivating Example

- ▶ Quasilinear utility: $u(x, y) = v(x) + y$
- ▶ Prices $(p, 1)$ and income m
- ▶ x is a *discrete good*: can only buy integer amounts
- ▶ y is continuous good
- ▶ If buy n units of good x , how many units of y can you buy?
units

3 / 26

How Many Units of Good 1?

- ▶ Suppose consumer is choosing bundle $(n, m - pn)$
- ▶ Must be preferred to bundle $(n + 1, m - p(n + 1))$
 - ▶ Must have

$$v(n) + m - np \geq v(n + 1) + m - (n + 1)p$$

- ▶ Rearrange: $p \geq v(n + 1) - v(n) \equiv r_{n+1}$
- ▶ Must be preferred to bundle $(n - 1, m - p(n - 1))$
 - ▶ Must have

$$v(n) + m - np \geq v(n - 1) + m - (n - 1)p$$

- ▶ Rearrange: $p \leq v(n) - v(n - 1) \equiv r_n$
- ▶ Will demand n units of good 1 iff

$$r_{n+1} \leq p \leq r_n$$

4 / 26

Reservation Prices

- ▶ Define *reservation prices*

$$r_0 = v(0)$$

$$r_1 = v(1) - v(0)$$

$$r_2 = v(2) - v(1)$$

...

$$r_n = v(n) - v(n-1)$$

- ▶ Prices at which consumer is just indifferent between consuming and not consuming additional unit

5 / 26

Gross and Net Surplus in Quasilinear Case

7 / 26

Consumer Surplus (Quasilinear Case)

- ▶ $v(n)$ is utility of consuming n units of good 1

- ▶ We call this the *gross consumer surplus*

- ▶ This is the area under the demand curve

- ▶ Set $v(0) = 0$

- ▶ Note that

$$v(n) = r(1) + r(2) + r(3) + \dots = \sum_{i=1}^n r_i$$

- ▶ If we subtract out the money spent on those n units, we get *net consumer surplus*:

$$v(n) - pn$$

- ▶ We often drop the “net” and just call this *consumer surplus* (CS)

- ▶ Intuition: extra happiness from consumption

6 / 26

Consumer Surplus (General Case)

- ▶ In general (ie continuous goods, any utility function), we define *consumer surplus* (CS) to be the area below the demand curve and above the price

8 / 26

Changes in Consumer Surplus

- ▶ Typically we care about the change in consumer surplus when price increases from p' to p''
- ▶ Let the demand be given by $x(p)$ (fixing income and all other prices)
- ▶ Then we can use an integral to calculate the *change in consumer surplus*:

$$\Delta CS = \int_{p''}^{p'} x(p) dp$$

- ▶ Note that integral is over p since demand is function of price
 - ▶ Usually graph with p on the vertical axis, so integral is along *vertical* axis

9 / 26

Changes in Consumer Surplus, Graphically

10 / 26

Example: Linear Demand

- ▶ Let demand be given by $x(p) = a - bp$
- ▶ Suppose price goes from 2 to 1
- ▶ What is change in consumer surplus?

Market Demand

11 / 26

12 / 26

Adding up Individual Demand

- ▶ Suppose there are N consumers in the market, indexed $i = 1, 2, \dots, N$
- ▶ Demand from consumer i : $x_i(p_1, p_2, m_i)$
- ▶ Define *aggregate* or market demand as

$$X(p_1, p_2, m_1, m_2, \dots, m_N) = \sum_{i=1}^N x_i(p_1, p_2, m_i)$$

- ▶ Assume that behavior of all agents together can be summarized by behavior of one agent with income equal to sum of all incomes
 - ▶ $M = \sum_{i=1}^N m_i$
 - ▶ This is called *representative consumer*
- ▶ Then market demand can be written $X(p_1, p_2, M)$
 - ▶ Focusing on one good and fixing all other prices and M , we also write $D(p)$ for the demand

13 / 26

Market Demand Curve

Adding Up Demand

- ▶ We can also think of price as a function of quantity, called *inverse demand*, which we write as $P(X)$ or $P(Q)$
- ▶ Demand and inverse demand are two ways of writing the same function
- ▶ However, we add up demands *horizontally* on the standard graph
- ▶ Two kinds of changes in purchase behavior:
 - ▶ Intensive margin: deciding how much to purchase
 - ▶ Extensive margin: deciding whether to purchase anything at all

15 / 26

Adding Up Demand: Example

14 / 26

16 / 26

Elasticity

- ▶ We want to know how responsive demand is to price
- ▶ We can look at the slope of the demand curve, $\frac{\Delta q}{\Delta p}$
- ▶ Problem: changing units makes comparing this number across markets difficult
- ▶ Solution: look at percent change in quantity per percent change in price:

$$\frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{p \Delta q}{q \Delta p}$$

- ▶ We call this number the *elasticity*, denoted by ϵ :

$$\epsilon(p) = \frac{p dq}{q dp}$$

- ▶ Note that we usually write elasticity as a function of price

17 / 26

Elasticity and Demand

- ▶ What sign will ϵ have?
- ▶ Usually we talk about magnitude of elasticity for convenience
- ▶ Three ranges:
 1. If $|\epsilon| > 1$, we have *elastic demand*
 - ▶ Quantity is very responsive to price
 - ▶ Change of price by 1% leads to change in demand of more than 1%
 - ▶ Examples?
 2. If $|\epsilon| < 1$, we have *inelastic demand*
 - ▶ Quantity is not very responsive to price
 - ▶ Change of price by 1% leads to change in demand of less than 1%
 - ▶ Examples?
 3. If $|\epsilon| = 1$, we say demand is *unit elastic*

18 / 26

Example: Linear Demand

- ▶ Let demand be given by $q = a - bp$
- ▶ What is elasticity of demand?
- ▶ Note that elasticity can change a lot with price
 - ▶ In example above, $|\epsilon|$ ranges from 0 to ∞

19 / 26

Elasticity of Linear Demand

20 / 26

Example: Constant Elasticity Demand

- ▶ Let demand be given by $q = Ap^k$
- ▶ What is the elasticity of demand?

Appendix

21 / 26

22 / 26

Compensating and Equivalent Variation

CV and EV

- ▶ Consider prices $(p_1^*, 1)$ and budget m^* , which leads to demand (x_1^*, x_2^*)
- ▶ Suppose we increase price of good 1 to \hat{p}_1 , so demand changes to (\hat{x}_1, \hat{x}_2)
- ▶ *Compensating variation* (CV): how far we have to shift new budget line to make consumer as well off as they were under original prices
- ▶ *Equivalent variation* (EV): how far we have to shift original budget line to make consumer as well off as they are under new prices

23 / 26

24 / 26

Example: Quasilinear Preferences

- ▶ Let $u(x_1, x_2) = v(x_1) + x_2$, $p_2 = 1$
- ▶ Suppose initially $p_1 = p_1^*$
 - ▶ Demand of good 1 is x_1^*
 - ▶ What is utility?
- ▶ Price increase to $p_1 = \hat{p}_1$
 - ▶ Demand of good 1 is \hat{x}_1
 - ▶ What is utility?
- ▶ What is compensating variation?

- ▶ What is equivalent variation?

- ▶ What is ΔCS ?

Relationship Between CV, EV, and ΔCS

- ▶ We saw that for quasilinear preferences $CV = \Delta CS = EV$
- ▶ In general, we have the following relationship between the three quantities:

$$CV \leq \Delta CS \leq EV$$

- ▶ This is why ΔCS is a good estimate of the welfare effects of changing prices