Econ 301: Microeconomic Analysis

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Technology

Context

- We have so far assumed supply function is given
- To determine supply function, we need to talk about production technology
  - Current technology defines what inputs are required to produce a given input
  - Or conversely, how much output is feasible from given inputs
- Producer theory has many analogies to consumer theory
  - Inputs are like consumption bundles
  - Output is like utility, though we can't rescale outputs as we could with utility

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- Usually measure inputs and outputs as flows, eg widgets per month

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- One option: list all the pairs (x, y) that are in the *production set*
  - x is the input and y is the output
  - We can draw the set of such pairs
- Another option: write out the *production function*
  - Really care only about maximum possible output for given input
  - This is the upper boundary of the production set
  - This defines *production function* $y = f(x)$ where x is input and y is output

Production Set and Production Function

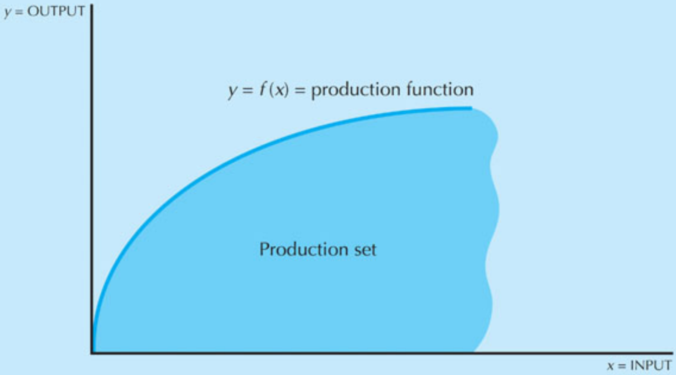

Figure 19.1 **A production set.** Here is a possible shape for a production set.

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- Analogous to indifference curves
  - Can't rescale isoquants like we did with indifference curves, however

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- Cobb-Douglas
  - Production function: $f(x_1, x_2) = Ax_1^a x_2^b$ where $A, a, b > 0$
  - A measures how many units of output we get for one unit each of inputs
  - Can't rescale exponents as in utility case

Properties of Production

Definition

A production technology is *monotonic* if $x_1 \geq z_1$ and $x_2 \geq z_2$ implies $f(x_1, x_2) \geq f(z_1, z_2)$. That is, output should increase if any inputs increase (and none decrease).

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Definition

Suppose $f(x_1, x_2) = f(z_1, z_2) = \bar{y}$. Then a production technology satisfies *convexity* if $f(\alpha x_1 + (1-\alpha)z_1, \alpha x_2 + (1-\alpha)z_2) \geq \bar{y}$. That is, a combination of two sets of inputs increases output.

Marginal Product

- How much more output do we get from adding more of input *i*?
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- How does MP_i change as x_i increases?
  - We know $MP_i > 0$ for all x_i because of monotonicity
  - We typically assume that production increases at a decreasing rate
  - This is know as *diminishing marginal product*
  - Formall, we have $\frac{\partial^2 f}{\partial x_i^2} < 0$
  - Analogy: diminishing marginal utility

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  - $TRS(x_1, x_2) = -\frac{MP_1}{MP_2}$
  - Analogy: marginal rate of substitution
- We typically assume that we have *diminishing TRS*
  - As we increase amount of factor 1, need less of a change in factor 2 to stay on isoquant
  - Analogy: diminishing marginal rate of substitution

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$$\frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 = 0$$

$$\frac{\partial f}{\partial x_2} dx_2 = -\frac{\partial f}{\partial x_1} dx_1$$

$$\frac{dx_2}{dx_1} = -\frac{\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial x_2}} = -\frac{MP_1}{MP_2}$$

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- Returns to scale of a given production function can be different at different productions levels

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- Let $f(x_1, x_2) = Ax_1^a x_2^b$
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  - Note that $f(kx_1, kx_2) = A(kx_1)^a(kx_2)^b = k^{a+b}Ax_1^a x_2^b = k^{a+b}f(x_1, x_2)$
  - Thus returns to scale are ...
    - increasing if $a + b > 1$
    - decreasing if $a + b < 1$
    - constant if $a + b = 1$

Long and Short Run

- In the *short run*, some inputs may be fixed at a certain level
- In the *long run*, all factors can be adjusted
- The length of these runs depends on context
- Example: farmer with fully-planted fields
    - Short run: land is fixed input
    - Long run: land can be bought and sold to re-optimize producer behavior

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- Which is the fixed factor in the short run? Typically capital