Econ 301: Microeconomic Analysis

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Game Applications

The Prisoner's Dilemma

- Recall Prisoner's Dilemma from last lecture:
  - Two suspects are being interrogated in two separate rooms
  - If they both Deny, go to jail for 2 years
  - If one Confesses, he gets 1 year while other gets 5
  - If they both Confess, go to jail for 4 years

	Deny	*Confess*
Deny	$(-2, -2)$	$(-5, -1)$
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- What will happen in this setting?

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  - All of our solution concepts agree that (Confess, Confess) is only reasonable outcome

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  - One possibility: repetition
  - Players may attempt to enforce cooperation by threatening with retaliation in future rounds
  - Let's see if this will work

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  - Thus unique backwards induction (or subgame perfect) equilibrium is for both players to play Confess each round
  - So repeating the game does not improve cooperation!

Game Theory with Firms

- Suppose we have a duopoly: just two firms producing in market
- Firms have just two strategies: pricing high or pricing low
- If both price high, split monopoly profits (3 each)
- If both price low, each gets competitive market profit of 2
- If one prices high and one prices low, profits are 1 for high price and 4 for low price firm

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- What is Nash equilbrium of this game?
  - Completely analogous to prisoner's dilemma
  - (Low, Low) is unique Nash equilibrium

Entry Deterrence

- Now suppose we have a monopoly, but a new firm is considering entering market
- Two players: incumbent and entrant
- Entrant chooses to enter or not enter (out)
- If Entrant does enter, monopolist can fight or allow
  - If fight, both firms get payoff 0
  - If allow, payoffs are 2 for entrant and 1 for incumbent
- If entrant does not enter, gets payoff 1 while incumbent gets payoff 3

Entry Deterrence (cont)

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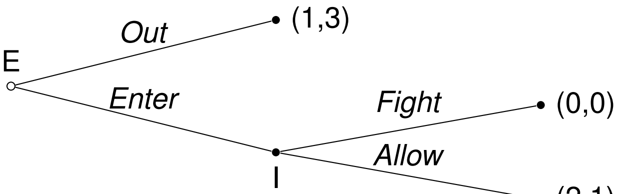

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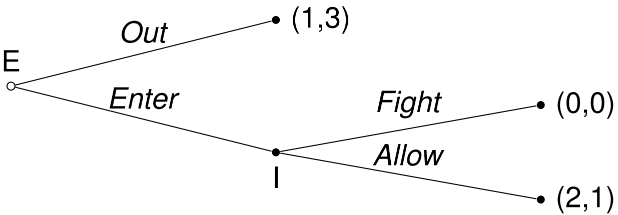

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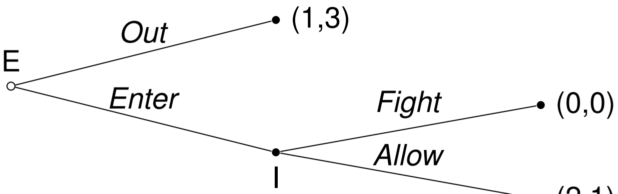

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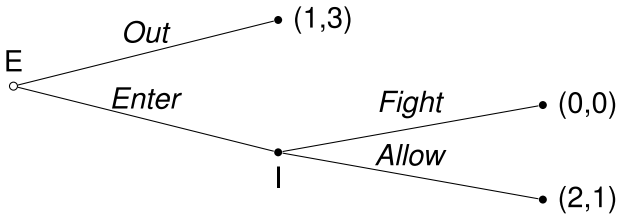

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  - In last round, incumbent will not fight, since $1 > 0$
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  - SPNE is (Enter, Allow)
- Note that incumbent prefers entrant play Out, but cannot make *credible threat* that he will fight if entrant enters

Making the Threat Credible

- Suppose that incumbent monopolist has previously invested in technology that allows it to better fight off competition
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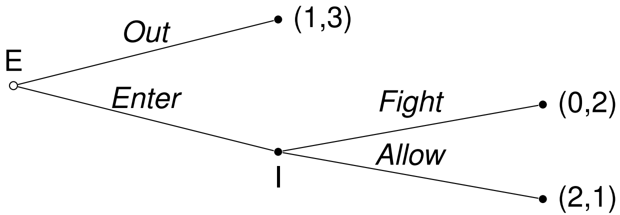

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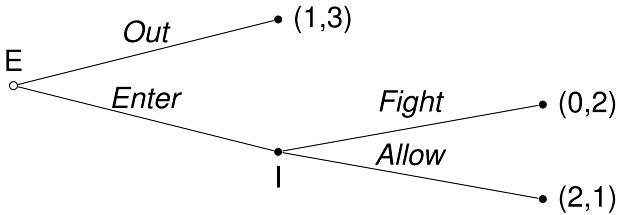

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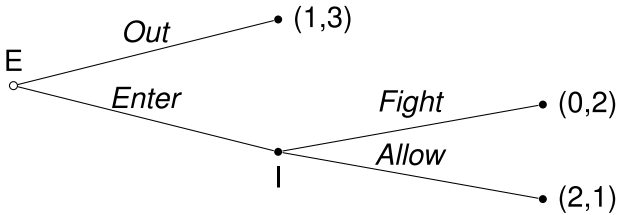

- What is SPNE now?
  - Incumbent will choose fight if entrant chooses to enter
  - Knowing this, entrant decides to stay out (since $1 > 0$)
  - SPNE is (Out, Fight)

Penalty Kicks

- Consider a game between a penalty kicker and a goalie in soccer
- Kicker can kick either left or right
- Goalie simultaneously decides whether to defend left or right
- Suppose kicker's accuracy is as follows:
  - 50% if kick left and goalie defends left
  - 80% if kick left and goalie defends right
  - 90% if kick right and goalie defends left
  - 20% if kick right and goalie defends right
- Assume kicker payoff is probability that she scores
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$$\begin{array}{ccc} & L & R \\ L & (50, -50) & (80, -80) \\ R & (90, -90) & (20, -20) \end{array}$$

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$$50q + 80(1-q) = 90q + 20(1-q)$$
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  - Goalie's indifference condition:

$$-50p - 90(1-p) = -80p - 20(1-p)$$
$$p = 0.7$$

  - Thus Nash equilibrium is $(p^*, q^*) = (0.7, 0.6)$