Econ 311: Behavioral and Experimental Economics

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Auctions

What is an Auction?

- An auction is a way of allocating a single good to one of many interested buyers
- Buyers typically submit *bids* which can be any positive number
- Who gets the item, and how much they pay for it, is a function of the all the bids
- Note that auctions are a type of game:
  - Bidders are the players
  - Bids are the strategies
  - Payoffs are winnings minus payments

When Are Auctions Used?

- Thin markets: unique/rare items such as art, collectibles
- Thick markets: common/mundane items such as fish, livestock, flowers, online ads
- Procurement: government seeking private companies to complete large projects
- Resources: government selling rights oil, land, bandwidth, pollution

Why Are Auctions Used so Frequently?

- Why do sellers like auctions?
  - Maximize revenue, since items are sold to buyer with highest value
- Why do buyers like auctions?
  - Transparency: simple rules, easy to verify bids
  - Fairness: anyone can participate
- Why do economists like auctions?
  - Efficient allocation of resources

Private vs Common Value

- Typically assume good being auctioned off has either *private value* or *common value*
  - Private value
    - Each bidder has different valuation of the good
    - Players do not know the values of others
    - Example? art auction
  - Common value
    - Each bidder has same value of good
    - Before submitting bid, bidders get a noisy signal of the value
    - Only find out the exact value after the good is allocated
    - Example? storage unit auctions

Different Ways to Run an Auction

- Ascending bid English auction: Bidders call out increasing prices until only one bidder is left
- Sealed bid auction: Bidders write down a bid, seal in envelope, and turn in to auctioneer
  - Winner is person who submits highest bid
  - Pays either their bid (*first price*) or the next-highest bid (*second price*)
- Descending bid Dutch auction: start with high price, lower until one person is willing to buy at that price
  - Called Dutch auction because it's how flowers are sold in Amsterdam

Optimal in Private Values English Ascending-Bid Auction

- Suppose you are in an English auction
- Strategy: point at which you will drop out of auction (call this your bid)
- Is bid above your valuation a good idea? No.
  - If you win item, you will get negative net payoff
  - So this strategy is dominated by one in which you bid a lower amount
- Is bid below your valuation a good idea? No.
  - Raising your drop-out point increases your chances of winning a net positive amount
  - So this bidding strategy is dominated by a higher one
- Thus the only strategy that is not dominated is when you bid your exact valuation

Optimal Behavior in Second-Price Sealed Bid Auction

- Suppose you are in a private values second price sealed bid auction
- Is bidding above your valuation a good idea? No.
  - If you win, you lose money
  - Thus this bid is dominated by a slightly lower one
- Is bidding below your valuation a good idea? No.
  - If you win, your bid was already above the second price, so no harm in bidding more
  - If you lost, in some cases you could have done better by raising your bid
  - Thus this bid is dominated by a slightly higher one
- The only strategy that is not (weakly) dominated is to bid your valuation
- Note that English auction and second-price sealed bid are *strategic equivalents*: they have same optimal strategy

Optimal Behavior in First-Price Sealed Bid Auction

- Suppose you are in a *first*-price sealed bid auction where bidders have private values
  - The price the winner will pay is their own bid
- Is bidding above your valuation a good idea?
  - No; if you win, you lose money on net
  - Better not to enter the auction at all
- Is bidding exactly your valuation a good idea?
  - No; if you win, you could almost certainly have lowered your price a little bit
- Optimal strategy is to *shade your bid*

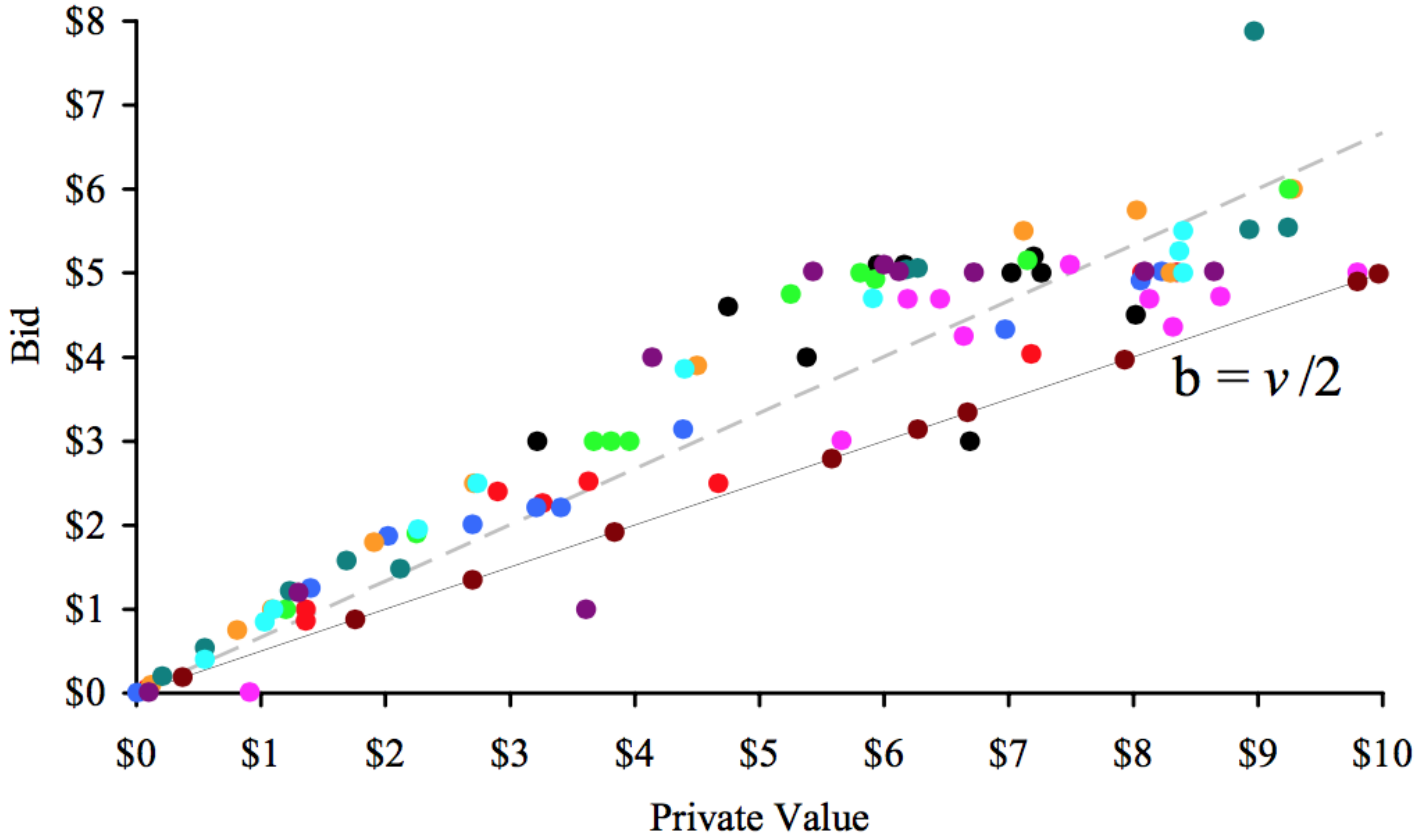
Formal Analysis for First-Price Sealed Bid

- Suppose you are bidding again a computer which will bid randomly between \$0 and \$10
- Your valuation: $v \in [\$0, \$10]$
- Your bid: b
- What is optimal bid b^* you should make?
  - Your payoff if you win with bid b: $v - b$
  - Probability that you win with this bid: $\frac{b}{10}$ (otherwise you get payoff 0)
  - Expected payoff (assume you are risk neutral): $(v - b)\frac{b}{10} = \frac{vb}{10} - \frac{b^2}{10}$
  - First order condition with respect to b: $\frac{v}{10} - \frac{2b}{10} = 0$
  - Solve to find $b^* = \frac{v}{2}$
- In general, if N people bidding, best response bid is $b^* = \frac{N-1}{N}v$

Formal Analysis, cont

- Now assume you are bidding against another player instead of computer
- Is both players bidding half their valuation a NE?
  - Suppose player $i = 1, 2$ has value $v_i \sim U[0, 10]$ and enters bid b_i
  - Suppose player 2 is indeed bidding $\frac{v_2}{2}$
  - Player 1's probability of winning is
    - 1 if $b_1 > 5$
    - $\frac{b_1}{5}$ if $b_1 \leq 5$
  - Expected payoff for 1 is then
    - $v_1 - b_1$ if $b_1 > 5$
    - $\frac{b_1}{5}(v_1 - b_1)$ if $b_1 \leq 5$
  - Can check this payoff is maximized at $b_1 = \frac{v_1}{2}$
  - Thus player is 1 best-responding
  - Proof for player 2 is symmetric
  - Thus this is a NE

Actual Behavior in Private Values First Price Auction



- Best fit line has slope of about $\frac{2}{3}$ (predicted should be $\frac{1}{2}$)

Source: Holt

Optimal Strategy in Private Values Dutch Auction

- Recall that in Dutch auction, value starts very high and drops until someone stops it
- Makes sense to pick a point at which you will stop the auction (if someone has not stopped it already)
- Does it make sense to have stopping point above your value? No
  - If you win, you will have negative net payoff
- Does it make sense to have stopping point exactly at your value? No
  - If you win, you will have zero net payoff
  - If you had waited another second, would have almost certainly won auction at better price
- Thus again you would want to shade your bid
- In fact, Dutch Auction and first-price sealed bid are *strategic equivalents*: optimal stopping point in Dutch auction is same as optimal bid in first-price auction

How do People Actually Bid in Auctions?

- In summary, we found:
  - In first-price sealed bid and descending clock, bidders should shade their bid (by $\frac{N-1}{N}$)
  - In second-price sealed bid and English, bidders should bid their valuation (ie no shading of bid)
- Do people actually play this way? Let's look at the data from our class exercise

Common Value Auctions

- Now move to common value auctions
  - Item is worth the same to everyone
  - Get a noisy signal of valuation before you buy
  - Find out exact value only after winner is announced
- Example: oil lease
  - Suppose gov't holds land that is worth v to gov't
  - Gov't decides to auction off land to oil companies
    - Companies can extract more value from land, say $1.5v$
    - Companies don't know v exactly; only get signal $v + \epsilon$
  - Winner is company that submit highest bid
  - This will be company that gets most optimistic signal
  - Winner company almost certainly will bid more than land is worth
  - This is called *winner's curse*