Econ 311: Behavioral and Experimental Economics

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Expected Utility: The Classic Theory

Motivating Example

- Suppose you are on the last round of the TV show *Who Wants to be a Millionaire*?
- You have narrowed down to two possible answers
  - Guess wrong: go home with \$32,000
  - Guess right: go home with \$1,000,000
- Walk away: go home with \$500,000 for certain
- What do you do?

Gambles

- We need a way to make choices between uncertain options, eg gambles
- Consider a gamble called A, for example
  - Possible outcomes are indexed by $i = 1, 2, 3, \ldots, n$
  - Probability of outcome i: p_i
  - Value of outcome i: x_i
  - Gamble is them summarized by $(p_1, x_1; p_2, x_2; \ldots; p_n, x_n)$

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  - Roll die, get paid the amount of the roll in dollars: $\left(\frac{1}{6}, \$1; \frac{1}{6}, \$2; \frac{1}{6}, \$3; \frac{1}{6}, \$4; \frac{1}{6}, \$5; \frac{1}{6}, \$6\right)$

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Expected Utility

- Expected utility
  - Consumer assigns utility $u(x)$ to wealth x
  - Expect utility theory says that

$$EU(A) = \sum_{i}^{n} p_i u(x_i) = p_1 u(x_1) + p_2 u(x_2) + \ldots + p_n u(x_n)$$

- Consumers will choose the gamble that maximizes expected utility

Why Is Expected Utility Reasonable?

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  4. Independence: For any gambles A, B, C such that $A \succeq B$ and any $p \in (0, 1]$, we must have $pA + (1 - p)C \succeq pB + (1 - p)C$.

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Theorem (von Nueman and Moregensten)

Preferences over gambles that satisfy conditions 1-4 can be represented by expected utility.

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- What is the intuition for this axiom?
  - How you feel about a prize (ie a specific amount of money) does not depend on the probability you receive it
- Example:
  - Suppose $A = (1, \$10)$, $B = \left(\frac{1}{2}, \$20; \frac{1}{2}, \$0\right)$, $C = (1, \$100)$
  - Suppose you like A more than B, ie $A \succeq B$
  - Then you must like

$$pA + (1 - p)C = (p, \$10; 1 - p, \$100)$$

more than

$$pB + (1 - p)C = \left(\frac{p}{2}, \$20; \frac{p}{2}, \$0; 1 - p, \$100\right)$$

What Shape Should $u(x)$ Have?

- Consider the following game: I will flip a coin until the first heads comes up. If the first heads is on flip number n, then I'll pay you \$$2^n$. How much would you pay to play this game?
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  - $EV = \frac{1}{2}2 + \frac{1}{4}4 + \frac{1}{8}8 + \ldots = 1 + 1 + 1 + \ldots = \infty$
- It is clear that there is a *diminishing marginal utility of money*
  - Intuition: an extra \$1000 is massive windfall for a very poor person but not even noticeable for very rich person
- We can rationalize the typically observed behavior by assuming that $u(x)$ is concave

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Risk Aversion vs Risk Seeking

- Can also have *risk-seeking* preferences (convex $u(x)$) where all of the above statements are reversed
- Can also have *risk-neutral* preferences (linear $u(x)$)

In summary:

Risk Averse	**Risk Neutral**	**Risk Seeking**
$u(x)$ concave	$u(x)$ linear	$u(x)$ convex
$EU(A) < u(EV(A))$	$EU(A) = u(EV(A))$	$EU(A) > u(EV(A))$
$RP > 0$	$RP = 0$	$RP < 0$

Risk Aversion: Example

- One family of utility functions take the form $u(x) = x^\alpha$
- In particular, let $u(x) = \sqrt{x}$, ie $\alpha = \frac{1}{2}$
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- Certainty equivalent of gamble: $\sqrt{CE} = 3.06 \rightarrow CE = 3.06^2 = \$9.36$
- Risk premium of gamble: $RP = \$10 - \$9.36 = \$0.64$

Lab Evidence: Holt and Laury (2002)

- 175 subjects from universities
- Ask to choose among options A and B for the following 10 cases:

TABLE 1—THE TEN PAIRED LOTTERY-CHOICE DECISIONS WITH LOW PAYOFFS

Option A	Option B	Expected payoff difference
1/10 of \$2.00, 9/10 of \$1.60	1/10 of \$3.85, 9/10 of \$0.10	\$1.17
2/10 of \$2.00, 8/10 of \$1.60	2/10 of \$3.85, 8/10 of \$0.10	\$0.83
3/10 of \$2.00, 7/10 of \$1.60	3/10 of \$3.85, 7/10 of \$0.10	\$0.50
4/10 of \$2.00, 6/10 of \$1.60	4/10 of \$3.85, 6/10 of \$0.10	\$0.16
5/10 of \$2.00, 5/10 of \$1.60	5/10 of \$3.85, 5/10 of \$0.10	−\$0.18
6/10 of \$2.00, 4/10 of \$1.60	6/10 of \$3.85, 4/10 of \$0.10	−\$0.51
7/10 of \$2.00, 3/10 of \$1.60	7/10 of \$3.85, 3/10 of \$0.10	−\$0.85
8/10 of \$2.00, 2/10 of \$1.60	8/10 of \$3.85, 2/10 of \$0.10	−\$1.18
9/10 of \$2.00, 1/10 of \$1.60	9/10 of \$3.85, 1/10 of \$0.10	−\$1.52
10/10 of \$2.00, 0/10 of \$1.60	10/10 of \$3.85, 0/10 of \$0.10	−\$1.85

- Repeated for 20x, 50x, 90x payoffs

Source: Holt and Laury (2002)

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- How should responses change with stakes?

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  3. Decreasing relative risk aversion: choices are less *risk* averse as stakes go up (i.e. switch earlier)

Results: Holt and Laury

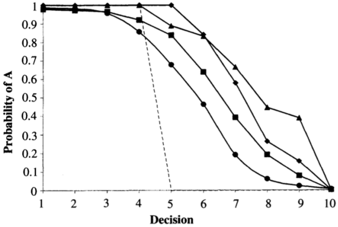

FIGURE 2. PROPORTION OF SAFE CHOICES IN EACH DECISION: DATA AVERAGES AND PREDICTIONS

Note: Data averages for low real payoffs [solid line with dots], 20x real [squares], 50x real [diamonds], 90x real payoffs [triangles], and risk-neutral prediction [dashed line].

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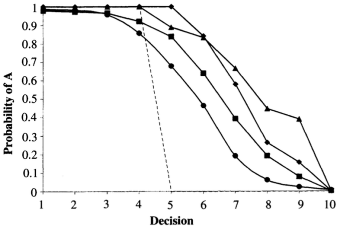

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  - Risk averse: note average switch point is well past row 5
- What is type of relative risk aversion?

Results: Holt and Laury

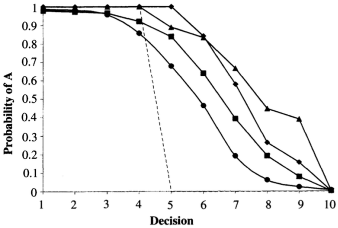

FIGURE 2. PROPORTION OF SAFE CHOICES IN EACH DECISION: DATA AVERAGES AND PREDICTIONS

Note: Data averages for low real payoffs [solid line with dots], 20x real [squares], 50x real [diamonds], 90x real payoffs [triangles], and risk-neutral prediction [dashed line].

- Is the average participant risk averse, risk neutral, or risk loving?
  - Risk averse: note average switch point is well past row 5
- What is type of relative risk aversion?
  - Increasing relative risk aversion: note lines move out as stakes increase

Source: Holt and Laury (2002)

Machina Triangles

- How do we graph risky prospects themselves?
- Suppose we fix payoff amounts $x_1 < x_2 < x_3$
- Let p_1, p_2, and p_3 vary
- Since $p_1 + p_2 + p_3 = 1$, really just two degrees of freedom
- Put p_1 on horizontal axis and p_3 on vertical axis
- Possible gambles lie in the triangle defined by $p_1 \geq 0$, $p_3 \geq 0$, and $p_1 + p_3 \leq 1$, hence the name *Machina triangle*

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    - x_1 for certain:

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  - x_3 for certain: $(0, 1)$
  - x_1 and x_2 with equal probability:

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  - x_1 and x_2 with equal probability: $(\frac{1}{2}, 0)$

Machina Triangles

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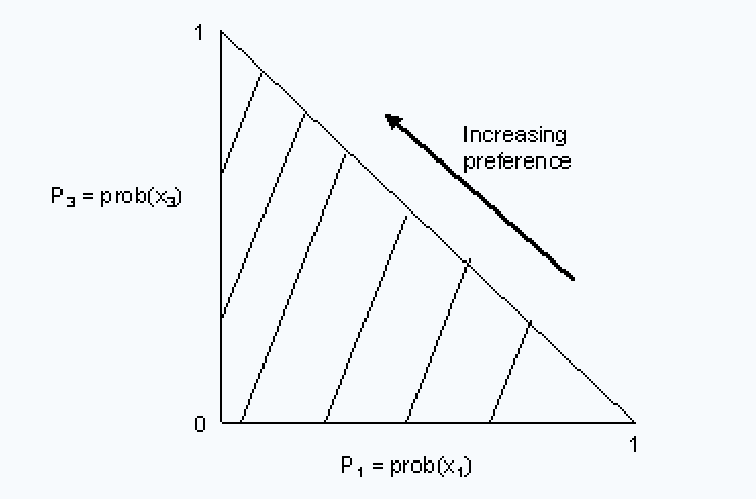
  - x_3 for certain: $(0, 1)$
  - x_1 and x_2 with equal probability: $(\frac{1}{2}, 0)$
  - x_1, x_2, and x_3 with equal probability: $(\frac{1}{3}, \frac{1}{3})$

Machina Triangle



Expected Utility in the Machina Triangle

- What do indifference curves in the Machina triangle look like for expected utility theory?

Expected Utility in the Machina Triangle

- What do indifference curves in the Machina triangle look like for expected utility theory?
- Set $EU = K$:

$$p_1 u(x_1) + (1 - p_1 - p_3) u(x_2) + p_3 u(x_3) = K$$

- Solve for p_3:

$$p_3 = \frac{u(x_2) - u(x_1)}{u(x_3) - u(x_2)} p_1 + C$$

- Indifference curves on Machina triangle are straight parallel lines with positive slope (increasing preference up and to the left)
- More risk aversion: steeper slope

The Allais Paradox: Common Consequence Version

1. Select your preferred option:
   A: Receive \$100 million for certain
   B: 10% chance of \$500 million, 89% chance of \$100 million, 1% chance of no money

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1. Select your preferred option:
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   B: 10% chance of \$500 million, 89% chance of \$100 million, 1% chance of no money
2. Select your preferred option:
   A': 11% chance of \$100 million, 89% chance of no money
   B': 10% chance of \$500 million, 90% chance of no money

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- $EU(A) =$

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- $EU(A) = u(100)$

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- $EU(A) = u(100)$
- $EU(B) = .1u(500) + .89u(100) + .01u(0)$

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- $EU(A') =$

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- $EU(B) = .1u(500) + .89u(100) + .01u(0)$
- $EU(A') = .11u(100) + .89u(0)$

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- $EU(A) = u(100)$
- $EU(B) = .1u(500) + .89u(100) + .01u(0)$
- $EU(A') = .11u(100) + .89u(0)$
- $EU(B') = .1u(500) + .9u(0)$

- Typical choice pattern?

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- $EU(A) = u(100)$
- $EU(B) = .1u(500) + .89u(100) + .01u(0)$
- $EU(A') = .11u(100) + .89u(0)$
- $EU(B') = .1u(500) + .9u(0)$

- Typical choice pattern? $A \succeq B$; $B' \succeq A'$

Allais Paradox, cont.

- Suppose you choose $A' \succeq B'$; can I predict your preference between A and B?

Allais Paradox, cont.

- Suppose you choose $A' \succeq B'$; can I predict your preference between A and B? Yes:

$$
\begin{aligned}
& EU(A') > EU(B') \\
& \implies .11u(100) + .89u(0) \geq .1u(500) + .9u(0) \\
& \implies .11u(100) + .89u(0) \geq .1u(500) + .89u(0) + .01u(0) \\
& \implies .11u(100) + .89u(100) \geq .1u(500) + .89u(100) + .01u(0) \\
& \implies u(100) \geq .1u(500) + .89u(100) + .01u(0) \\
& \implies EU(A) > EU(B)
\end{aligned}
$$

- Typical choice pattern is incompatible with expected utility theory
- Called *common consequence* version of the Allais Paradox, because I added the .89 chance of \$100 million to both sides

The Allais Paradox: Common Ratio Version

1. Select your preferred option:
   C: Receive \$100 million for certain
   D: 98% chance of \$500 million, 2% chance of no money
2. Select your preferred option:
   C': 1% chance of \$100 million, 99% chance of no money
   D': 0.98% chance of \$500 million, 99.02% chance of no money

- $EU(C) =$

The Allais Paradox: Common Ratio Version

1. Select your preferred option:
   C: Receive \$100 million for certain
   D: 98% chance of \$500 million, 2% chance of no money
2. Select your preferred option:
   C': 1% chance of \$100 million, 99% chance of no money
   D': 0.98% chance of \$500 million, 99.02% chance of no money

- $EU(C) = u(100)$

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   D': 0.98% chance of \$500 million, 99.02% chance of no money

- $EU(C) = u(100)$
- $EU(D) =$

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   D: 98% chance of \$500 million, 2% chance of no money
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   D': 0.98% chance of \$500 million, 99.02% chance of no money

- $EU(C) = u(100)$
- $EU(D) = .98u(500) + .02u(0)$

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   D': 0.98% chance of \$500 million, 99.02% chance of no money

- $EU(C) = u(100)$
- $EU(D) = .98u(500) + .02u(0)$
- $EU(C') =$

The Allais Paradox: Common Ratio Version

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   C: Receive \$100 million for certain
   D: 98% chance of \$500 million, 2% chance of no money
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   D': 0.98% chance of \$500 million, 99.02% chance of no money

- $EU(C) = u(100)$
- $EU(D) = .98u(500) + .02u(0)$
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- $EU(C) = u(100)$
- $EU(D) = .98u(500) + .02u(0)$
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- $EU(D') =$

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- $EU(C) = u(100)$
- $EU(D) = .98u(500) + .02u(0)$
- $EU(C') = .01u(100) + .99u(0)$
- $EU(D') = .0098u(500) + .9902u(0)$

- Typical choice pattern?

The Allais Paradox: Common Ratio Version

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- $EU(C) = u(100)$
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- $EU(C') = .01u(100) + .99u(0)$
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- Typical choice pattern? $C \succeq D$; $D' \succeq C'$

Allais Paradox, cont.

- Suppose you choose $C \succeq D$; can I predict your preference between C' and D'?

Allais Paradox, cont.

- Suppose you choose $C \succeq D$; can I predict your preference between C' and D'? Yes:

$$
\begin{aligned}
&EU(C) > EU(D) \\
&\implies u(100) \geq .98u(500) + .02u(0) \\
&\implies 0.01u(100) \geq .0098u(500) + .0002u(0) \\
&\implies 0.01u(100) + 0.99u(0) \geq .0098u(500) + .0002u(0) + 0.99u(0) \\
&\implies 0.01u(100) + 0.99u(0) \geq .0098u(500) + .9902u(0) \\
&\implies EU(C') > EU(D')
\end{aligned}
$$

- Typical choice pattern is incompatible with expected utility theory
- Called *common ratio* version of the Allais Paradox, because I multipled both sides of the equation by 0.01

What Is Going On?

- Expected utility theory says we should have $A \succeq B \iff A' \succeq B'$ and $C \succeq D \iff C' \succeq D'$
  - But many responses violate these assertions
- Both of these assertions come from independence axiom
- So if revealed preferences don't follow these results, EUT must not represent people's true preferences
- That is, we need another theory that does not have independence but better explains behavior
  - This is where we will go for next class