Econ 311: Behavioral and Experimental Economics

Prof. Jeffrey Naecker

Wesleyan University

Auctions

What is an Auction?

- An auction is a way of allocating a single good to one of many interested buyers
- Buyers typically submit *bids* which can be any positive number
- Who gets the item, and how much they pay for it, is a function of the all the bids

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- Note that auctions are a type of game:
  - Bidders are the players
  - Bids are the strategies
  - Payoffs are winnings minus payments

When Are Auctions Used?

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- Thin markets: unique/rare items such as art, collectibles
- Thick markets: common/mundane items such as fish, livestock, flowers, online ads
- Procurement: government seeking private companies to complete large projects
- Resources: government selling rights oil, land, bandwidth, pollution

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- Why do economists like auctions?
  - Efficient allocation of resources

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  - Pays either their bid (*first price*) or the next-highest bid (*second price*)
- Descending bid Dutch auction: start with high price, lower until one person is willing to buy at that price
  - Called Dutch auction because it's how flowers are sold in Amsterdam

Optimal in Private Values English Ascending-Bid Auction

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- Thus the only strategy that is not dominated is when you bid your exact valuation

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- The only strategy that is not (weakly) dominated is to bid your valuation
- Note that English auction and second-price sealed bid are *strategic equivalents*: they have same optimal strategy

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- Optimal strategy is to *shade your bid*

Formal Analysis for First-Price Sealed Bid

- Suppose you are bidding again a computer which will bid randomly between \$0 and \$10
- Your valuation: $v \in [\$0, \$10]$
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  - Solve to find $b^* = \frac{v}{2}$
- In general, if N people bidding, best response bid is $b^* = \frac{N-1}{N}v$

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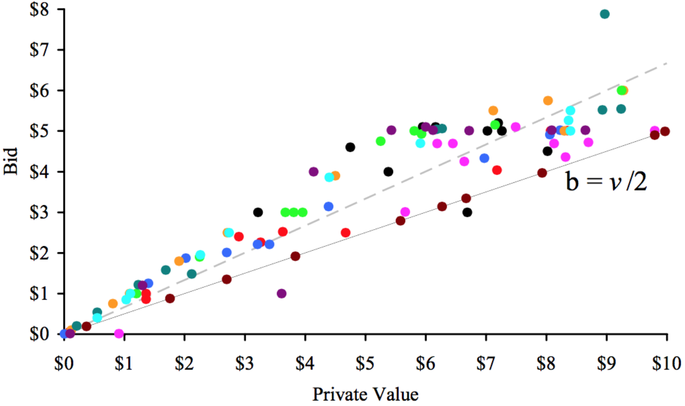
  - Thus player is 1 best-responding
  - Proof for player 2 is symmetric
  - Thus this is a NE

Actual Behavior in Private Values First Price Auction



- Best fit line has slope of about $\frac{2}{3}$ (predicted should be $\frac{1}{2}$)

Source: Holt

Optimal Strategy in Private Values Dutch Auction

- Recall that in Dutch auction, value starts very high and drops until someone stops it
- Makes sense to pick a point at which you will stop the auction (if someone has not stopped it already)
- Does it make sense to have stopping point above your value?

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- Thus again you would want to shade your bid
- In fact, Dutch Auction and first-price sealed bid are *strategic equivalents*: optimal stopping point in Dutch auction is same as optimal bid in first-price auction

How do People Actually Bid in Auctions?

- In summary, we found:
  - In first-price sealed bid and descending clock, bidders should shade their bid (by $\frac{N-1}{N}$)
  - In second-price sealed bid and English, bidders should bid their valuation (ie no shading of bid)
- Do people actually play this way?

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  - In second-price sealed bid and English, bidders should bid their valuation (ie no shading of bid)
- Do people actually play this way? Let's look at the data from our class exercise

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- Example: oil lease
  - Suppose gov't holds land that is worth v to gov't
  - Gov't decides to auction off land to oil companies
    - Companies can extract more value from land, say $1.5v$
    - Companies don't know v exactly; only get signal $v + \epsilon$
  - Winner is company that submit highest bid
  - This will be company that gets most optimistic signal
  - Winner company almost certainly will bid more than land is worth
  - This is called *winner's curse*