

Econ 311: Behavioral and Experimental Economics

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Review of Game Theory

Motivation

- ▶ We need a tool for analyzing behavior when we have more than one decision-maker
- ▶ In many cases, we can assume competitive markets with large numbers of decision-makers
 - ▶ No one agent has a noticeable impact on the outcome
- ▶ However, we often end up in situations where the typical market assumptions do not hold
- ▶ This is where game theory becomes useful

What is a Game?

- ▶ First, we have several building blocks:
 - ▶ A *player* is a decision-maker in the game
 - ▶ A *strategy* is a complete contingent plan that a player makes for every possible point in the game where she can make a decision
 - ▶ A *payoff function* tells us what the winnings of each player will be as a function of all their strategies
- ▶ A game (in normal form) is a set of players, a set of possible strategies for those players, and a payoff function

Example of a Game

- ▶ We can represent a normal-form game with a matrix
 - ▶ Rows indicate strategies for player 1
 - ▶ Columns indicate strategies for player 2
 - ▶ Cells show payoffs for the two players
 - ▶ Usually put player 1 (row player) payoffs first in list

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 - ▶ Usually put player 1 (row player) payoffs first in list
- ▶ For example, a famous game called the Prisoner's Dilemma
 - ▶ Players can either cooperate (C) or defect (D)
- ▶ Payoff matrix:

	<i>C</i>	<i>D</i>
<i>C</i>	$(-2, -2)$	$(-5, -1)$
<i>D</i>	$(-1, -5)$	$(-4, -4)$

Solution Concepts

- ▶ A *solution concept* is a rule that, given any game, predicts which outcome(s) will actually happen when people play the game
- ▶ Focus on two solution concepts from classic game theory:
 - ▶ Nash Equilibrium
 - ▶ Dominant Strategies
 - ▶ Dominated Strategies
 - ▶ Subgame Perfect Nash Equilibrium (backwards induction)

Nash Equilibrium

- ▶ Strategies for row player: r_1, r_2, r_3, \dots
- ▶ Strategies for column player: c_1, c_2, c_3, \dots
- ▶ Let $BR_r(c)$ be the row player's best response function
 - ▶ That is, if column player is playing c , row player can maximize payoff by playing $BR_r(c)$
- ▶ Similarly, let $BR_c(r)$ be the column player's best response function

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Definition

The strategies r^{NE}, c^{NE} are a *Nash Equilibrium* if

$$r^{NE} = BR_r(c^{NE}) \quad \text{and} \quad c^{NE} = BR_c(r^{NE}).$$

- ▶ That is, both players are best-responding to each other
- ▶ Check NE by ensuring that no player has incentive to deviate

Nash Equilibrium of Prisoner's Dilemma

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- ▶ What is the Nash equilibrium of the Prisoner's Dilemma?
 - ▶ If your opponent is choosing cooperate, your best response to choose defect, since $(-1 > -2)$
 - ▶ If your opponent is choosing defect, your best response to choose defect, since $(-4 > -5)$
 - ▶ Thus NE is that both players defect

Dominant Strategies

- ▶ The strategy r^D is a *dominant strategy* iff

$$r^D = BR_r(c) \quad \text{for all } c = c_1, c_2, c_3, \dots$$

- ▶ That is, r^D is *always* the row player's best response, regardless of what the column player is doing
- ▶ Definition is similar for column player
- ▶ If both players have a dominant strategy, then the game has a *dominant strategy solution*

Dominated Strategies

- ▶ A strategy is *dominated* if it is *never* the best response for a player
- ▶ This gives us another solution concept: players will not play dominated strategies
- ▶ Relation to dominant strategies:
 - ▶ Possible to have strategies that are neither dominant nor dominated
 - ▶ In simple 2-by-2 games: if one strategy is dominant, other will be dominated
 - ▶ In more complex games: possible to have strategies that are dominated even if there is not dominant strategy

Prisoner's Dilemma

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- ▶ Does the Prisoner's dilemma have any dominant or dominated strategies?
 - ▶ Defect is a dominant strategy for both players
 - ▶ Cooperate is a dominated strategy for both players
 - ▶ Thus the only possible outcome is (Defect, Defect)

Common Knowledge of Rationality

- ▶ Note that Nash Equilibrium has a key assumption built in
 - ▶ Players must assume that all other players are capable of calculating their best response
 - ▶ Must assume that all other players know that they know this
 - ▶ And that all player know that they know that they know this
 - ▶ And so on ...

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- ▶ Dominant/dominated strategies assume less about the other players, so don't need common knowledge
 - ▶ But as a result, dominant/dominated strategies will in general make less specific predictions about outcomes of a game
 - ▶ Dominant/dominated strategies may not exist at all, in fact, in which case that concept makes no prediction at all

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- ▶ Is the assumption of common knowledge of rationality a good assumption for human behavior?

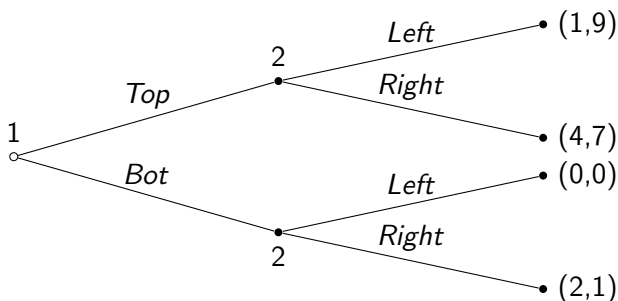
Sequential Games

- ▶ Consider the following game
 - ▶ Player 1 chooses Top or Bottom
 - ▶ Observing 1's choice, player 2 then chooses Left or Right
- ▶ This is a *sequential game*, because players move in sequence rather than simultaneously
- ▶ Payoff function:

(Top, Left)	→ (1, 9)
(Top, Right)	→ (4, 7)
(Bottom, Left)	→ (0, 0)
(Bottom, Right)	→ (2, 1)
- ▶ Note Player 2 really now has more complicated strategies, since must pick what to do after each move by player 1

Extensive Form

- ▶ We analyze such games in *extensive form* with a game tree:



- ▶ Note that sequential form has:
 - ▶ Every non-terminal node labeled with player who moves at that point
 - ▶ Every terminal node labeled with payoffs
 - ▶ Every branch labeled with available actions

Solution Concept: Subgame Perfect Nash Equilibrium

- ▶ We solve extensive form games with *backwards induction*
 - ▶ Start with end of the game tree
 - ▶ Determine what last mover will do
 - ▶ Take one step backwards in tree and repeat until all decisions have been analyzed

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- ▶ The solution we arrive at is called the *subgame perfect Nash equilibrium*
- ▶ Note that in sequential games, strategies must list action at every node at which the player moves
 - ▶ For example, player 2's strategy must indicate what 2 will do if 1 plays Top *and* what 2 will do if 1 plays Bottom
 - ▶ Notation: *RL* means play Right if Top, Left if Bottom, for example

Example

- ▶ What is backwards induction solution to game on previous slide?

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 - ▶ After Top, player 2 will play Left
 - ▶ After Bottom, player 2 will play Right
 - ▶ Given what player 2 will do, player 1 will choose to play Bottom
 - ▶ SPNE strategies are (B, LR)
 - ▶ SPNE outcome is (Bottom, Right)

Behavioral Game Theory

Motivating Example

- ▶ Nagel (1995) examines *beauty contest game*, also known as *guessing game*
 - ▶ Large number of players M
 - ▶ Positive number p is told to players (assume $2p \leq M$)
 - ▶ Each player picks a number from 0 to 100
 - ▶ Average guess X is calculated
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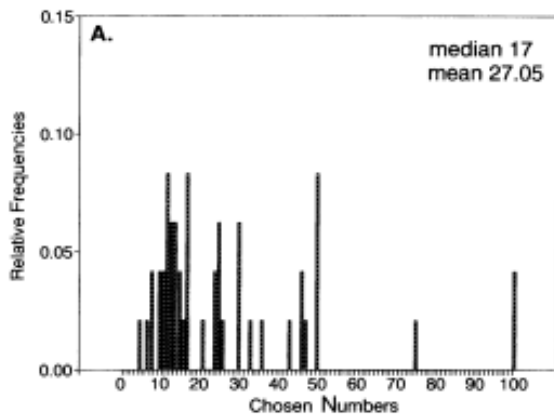
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 - ▶ If $p < 1$, any guess above $100p$ is dominated
 - ▶ No dominant strategies

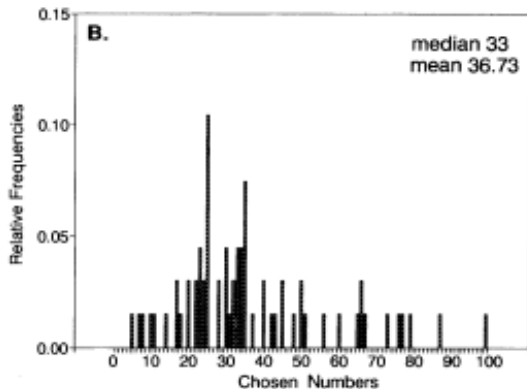
How Do People Actually Play?

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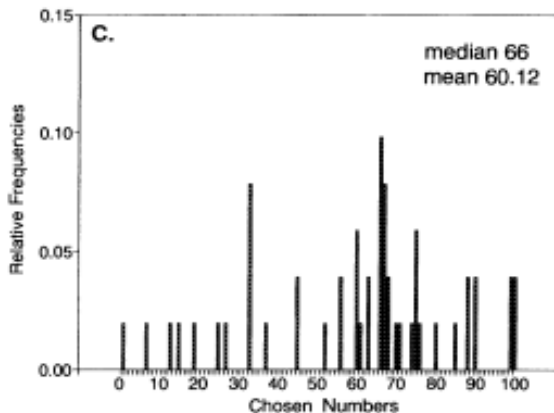
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How Do People Actually Play?

$$p = \frac{4}{3}$$



Failure of Standard Solution Concepts

- ▶ Clearly Nash equilibrium does not make good prediction
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- ▶ Clearly Nash equilibrium does not make good prediction
- ▶ Many players even choose dominated strategies
- ▶ Yet clearly subjects are not playing randomly, so what is going on?
 - ▶ Playing dominated strategy is difficult to justify
 - ▶ But playing other non-NE strategies are reasonable if we don't have common knowledge of rationality

Iterative Thinking

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 - ▶ Thus we should play $12.5p = 6.25$

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 - ▶ Thus we should play $12.5p = 6.25$
- ▶ We can repeat ad infinitum
- ▶ Note that we can keep doing this until we hit 0

New Solution Concept: Level k

- ▶ Define recursive set of strategies:
 - ▶ Level 0: Naive (non-strategic) play
 - ▶ Usually guessing randomly, but other assumptions make sense in other games
 - ▶ Level 1: best-respond to level 0
 - ▶ Level 2: best-respond to level 1 . . .
 - ▶ Level k : best-respond to level $k - 1$
- ▶ New solution concept: players will select one of the level k strategies, typically for $k = 1, 2$, or 3

Predictions of Level- k

- ▶ Note that level- k predicts we should see behavior cluster at 50, $50p$, $50p^2$, and so on
- ▶ Examining the data from Nagel (1995), we see
 - ▶ $p = \frac{1}{2}$: clusters at 50, 25, 12.5
 - ▶ $p = \frac{2}{3}$: clusters at 33, 22
 - ▶ Also note cluster at 67: confusion about rules?
 - ▶ $p = \frac{4}{3}$: clusters at 67, 88

Another Kind of Guessing Game

- ▶ Suppose you are playing with a partner
- ▶ You and your partner both submit guesses between 1 and 19
- ▶ Your payoffs:
 - ▶ Always get your guess in dollars
 - ▶ If your guess is exactly 3 less than opponent's guess, you get an additional bonus of 50 dollars
 - ▶ If your guess is exactly equal to opponent's guess, you get an additional bonus of 25 dollars

Analysis

- ▶ What is L_0 guess?

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- ▶ What is/are NE?

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- ▶ L_1 ?
 - ▶ Best response to 19 is to guess 16
- ▶ L_2 ?
 - ▶ Best response to 16 is to guess 13
- ▶ What is/are NE?
 - ▶ Level k converges towards guessing 1
 - ▶ If both guess 1, neither player can profitably deviate
 - ▶ Getting payoff $1 + 25 = 26$
 - ▶ Raising guess to x will get payoff $x \leq 19$

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 - ▶ Thus NE are $(1, 1)$, $(2, 2)$, and $(3, 3)$
- ▶ Are any guesses dominant or dominated?
 - ▶ Guessing 17, 18 or 19 dominated
 - ▶ No dominant strategies

Summary

- ▶ Solution concepts make predictions about what strategies will be played:
 - ▶ Nash Equilibrium: mutual best response
 - ▶ Dominant strategy: always a best response
 - ▶ Dominated strategy: never a best response
 - ▶ Level k : iterative best responses
- ▶ In experiments, we see that people
 - ▶ do not always play Nash
 - ▶ sometimes choose dominated strategies
 - ▶ often play $L1$, $L2$, or $L3$