Econ 311: Behavioral and Experimental Economics

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Review of Game Theory

Motivation

- We need a tool for analyzing behavior when we have more than one decision-maker
- In many cases, we can assume competitive markets with large numbers of decision-makers
  - No one agent has a noticeable impact on the outcome
- However, we often end up in situations where the typical market assumptions do not hold
- This is where game theory becomes useful

What is a Game?

- First, we have several building blocks:
  - A *player* is a decision-maker in the game
  - A *strategy* is a complete contingent plan that a player makes for every possible point in the game where she can make a decision
  - A *payoff function* tells us what the winnings of each player will be as a function of all their strategies
- A game (in normal form) is a set of players, a set of possible strategies for those players, and a payoff function

Example of a Game

- We can represent a normal-form game with a matrix
  - Rows indicate strategies for player 1
  - Columns indicate strategies for player 2
  - Cells show payoffs for the two players
    - Usually put player 1 (row player) payoffs first in list
- For example, a famous game called the Prisoner's Dilemma
  - Players can either cooperate (C) or defect (D)
- Payoff matrix:

	C	D
C	$(-2, -2)$	$(-5, -1)$
D	$(-1, -5)$	$(-4, -4)$

Solution Concepts

- A *solution concept* is a rule that, given any game, predicts which outcome(s) will actually happen when people play the game
- Focus on two solution concepts from classic game theory:
  - Nash Equilibrium
  - Dominant Strategies
  - Dominated Strategies
  - Subgame Perfect Nash Equilibrium (backwards induction)

Nash Equilibrium

- Strategies for row player: $r_1, r_2, r_3, \ldots$
- Strategies for column player: $c_1, c_2, c_3, \ldots$
- Let $BR_r(c)$ be the row player's best response function
  - That is, if column player is playing c, row player can maximize payoff by playing $BR_s(c)$
- Similarly, let $BR_c(r)$ be the column player's best response function

Definition

The strategies r^{NE}, c^{NE} are a *Nash Equilibrium* if

$$r^{NE} = BR_r(c^{NE}) \quad \text{and} \quad c^{NE} = BR_c(r^{NE}).$$

  - That is, both players are best-responding to each other
  - Check NE by ensuring that no player has incentive to deviate

Nash Equilibrium of Prisoner's Dilemma

	C	D
C	$(-2, -2)$	$(-5, -1)$
D	$(-1, -5)$	$(-4, -4)$

- What is the Nash equilibrium of the Prisoner's Dilemma?

Dominant Strategies

- The strategy r^D is a *dominant strategy* iff

$$r^D = BR_r(c) \quad \text{for all } c = c_1, c_2, c_3, \ldots.$$

- That is, r^D is *always* the row player's best response, regardless of what the column player is doing
- Definition is similar for column player
- If both players have a dominant strategy, then the game has a *dominant strategy solution*

Dominated Strategies

- A strategy is *dominated* if it is *never* the best response for a player
- This gives us another solution concept: players will not play dominated strategies
- Relation to dominant strategies:
  - Possible to have strategies that are neither dominant nor dominated
  - In simple 2-by-2 games: if one strategy is dominant, other will be dominated
  - In more complex games: possible to have strategies that are dominated even if there is not dominant strategy

Prisoner's Dilemma

	C	D
C	$(-2,-2)$	$(-5,-1)$
D	$(-1,-5)$	$(-4,-4)$

- Does the Prisoner's dilemma have any dominant or dominated strategies?

Common Knowledge of Rationality

- Note that Nash Equilibrium has a key assumption built in
  - Players must assume that all other players are capable of calculating their best response
  - Must assume that all other players know that they know this
  - And that all player know that they know that they know this
  - And so on ...
  - This is called *common knowledge of rationality*
- Dominant/dominated strategies assume less about the other players, so don't need common knowledge
  - But as a result, dominant/dominated strategies will in general make less specific predictions about outcomes of a game
  - Dominant/dominated strategies may not exist at all, in fact, in which case that concept makes no prediction at all
- Is the assumption of common knowledge of rationality a good assumption for human behavior?

Sequential Games

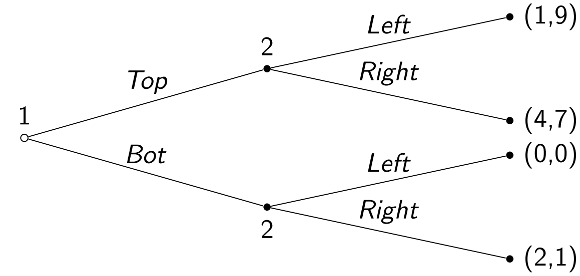
- Consider the following game
  - Player 1 chooses Top or Bottom
  - Observing 1's choice, player 2 then chooses Left or Right
- This is a *sequential game*, because players move in sequence rather than simultaneously
- Payoff function:

$$\begin{array}{ll} \text{(Top, Left)} & \to (1,9) \\ \text{(Top, Right)} & \to (4,7) \\ \text{(Bottom, Left)} & \to (0,0) \\ \text{(Bottom, Right)} & \to (2,1) \end{array}$$

- Note Player 2 really now has more complicated strategies, since must pick what to do after each move by player 1

Extensive Form

- We analyze such games in *extensive form* with a game tree:



- Note that sequential form has:
  - Every non-terminal node labeled with player who moves at that point
  - Every terminal node labeled with payoffs
  - Every branch labeled with available actions

Solution Concept: Subgame Perfect Nash Equilibrium

- We solve extensive form games with *backwards induction*
  - Start with end of the game tree
  - Determine what last mover will do
  - Take one step backwards in tree and repeat until all decisions have been analyzed
- The solution we arrive at is called the *subgame perfect Nash equilibrium*
- Note that in sequential games, strategies must list action at every node at which the player moves
  - For example, player 2's strategy must indicate what 2 will do if 1plays Top *and* what 2 will do if 1 plays Top
  - Notation: *RL* means play Right if Top, Left if Bottom, for example

Example

- What is backwards induction solution to game on previous slide?

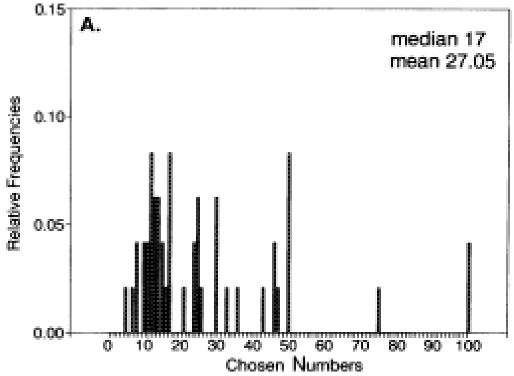
Behavioral Game Theory

Motivating Example

- Nagel (1995) examines *beauty contest game*, also known as *guessing game*
  - Large number of players M
  - Positive number p is told to players (assume $2p \leq M$)
  - Each player picks a number from 0 to 100
  - Average guess X is calculated
  - Player closest to pX wins a prize
- What are the NE of this game?
- What are the dominant/dominated strategies in this game?

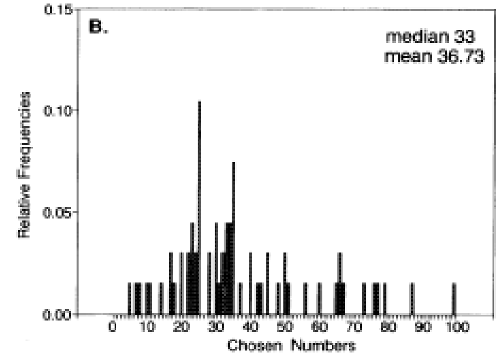
How Do People Actually Play?

$p = \frac{1}{2}$



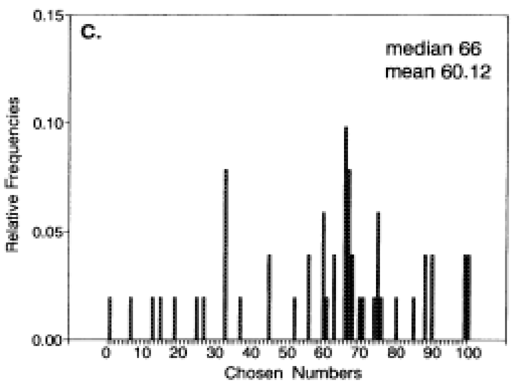
How Do People Actually Play?

$p = \frac{2}{3}$



How Do People Actually Play?

$p = \frac{4}{3}$



Failure of Standard Solution Concepts

- Clearly Nash equilibrium does not make good prediction
- Many players even choose dominated strategies
- Yet clearly subjects are not playing randomly, so what is going on?

Iterative Thinking

- For illustration, let $p = \frac{1}{2}$
- Suppose that we believe all other players to completely naive, and thus guessing randomly. What should we guess?
- However, we might realize that the other players will see this logic and play 25 themselves. Then what should we guess?
- We might then realize that the other players will see this logic and play 12.5 themselves. Then what should we guess?
- We can repeat ad infinitum
- Note that we can keep doing this until we hit 0

New Solution Concept: Level k

- Define recursive set of strategies:
  - Level 0: Naive (non-strategic) play
    - Usually guessing randomly, but other assumptions make sense in other games
  - Level 1: best-respond to level 0
  - Level 2: best-respond to level 1 . . .
  - Level k: best-respond to level $k-1$
- New solution concept: players will select one of the level k strategies, typically for $k = 1, 2$, or 3

Predictions of Level-k

- Note that level-k predicts we should see behavior cluster at 50, $50p$, $50p^2$, and so on
- Examining the data from Nagel (1995), we see
  - $p = \frac{1}{2}$: clusters at 50, 25, 12.5
  - $p = \frac{2}{3}$: clusters at 33, 22
    - Also note cluster at 67: confusion about rules?
  - $p = \frac{4}{3}$: clusters at 67, 88

Another Kind of Guessing Game

- Suppose you are playing with a partner
- You and your partner both submit guesses between 1 and 19
- Your payoffs:
  - Always get your guess in dollars
  - If your guess is exactly 3 less than opponent's guess, you get an additional bonus of 50 dollars
  - If your guess is exactly equal to opponent's guess, you get an additional bonus of 25 dollars

Analysis

- What is $L0$ guess?
- $L1$?
- $L2$?
- What is/are NE?
- Are any guesses dominant or dominated?

Summary

- Solution concepts make predictions about what strategies will be played:
  - Nash Equilibrium: mutual best response
  - Dominant strategy: always a best response
  - Dominated strategy: never a best response
  - Level k: iterative best responses
- In experiments, we see that people
  - do not always play Nash
  - sometimes choose dominated strategies
  - often play $L1$, $L2$, or $L3$